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Probabilistic Seismic Hazard Analysis: Problem and Correction

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Abstract: Probabilistic seismic hazard analysis (PSHA), the most widely used method for assessing seismic hazard and risk, contains an error in its hazard calculation: incorrectly equating the conditional exceedance probability of the ground-motion attenuation relationship (a function) to the exceedance probability of the ground-motion uncertainty (a variable). This error results in using the ground-motion uncertainty (spatial characteristic) to extrapolate occurrence of ground motion (temporal characteristic) or the *ergodic* assumption. This error also results in difficulty in understanding and applying PSHA. An alternative approach, called KY-PSHA, is developed to correct the error in this paper.

Key words: **Probabilistic seismic hazard analysis; Grand-motion; Ground-motion attenuation relationship; Ground-motion uncertainty**

地震危险性概率分析方法:存在的问题和纠正

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摘 要:地震危险性概率分析(PSHA)是目前最广泛应用于地震灾害与风险性评估的方法。然而它在计算中却存在着一个错误:把强地面运动衰减关系(一个函数)的条件超越概率等同于强地面运动误差(一个变量)的超越概率。这个错误导致了运用强地面运动误差(空间分布特征)去外推强地面运动的发生(时间分布特征)或称之为遍历性假设,同时也造成了对PSHA理解和应用上的困难。本文推导出新的灾害计算方法(称之为KY-PSHA)来纠正这种错误。

关键词:地震危险性概率分析;强地面运动;强地面运动衰减关系;强地面运动误差(不确定性)
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0 Introduction

Probabilistic seismic hazard analysis (PSHA), initially introduced by Cornell [1], has become the most commonly used method to assess seismic hazard and risk. The U. S. Geological Survey used PSHA to develop national seismic hazard maps[2-4]. These maps are the basis for national seismic safety regulations and design standards, such as the NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures[5-6], the 2000 International Building Code[7], and the 2000 International Residential Code[8]. Seismic design parameters for critical facilities, such as nuclear power plants, are also determined using PSHA[9]. The use of PSHA has caused great difficulty in terms of selecting a hazard or risk level for engineering design and other policy applications, however[10-23]. For example, an unphysically high ground motion of 10g PGA or greater has to be considered for a nuclear waste re-

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pository in Yucca Mountain, Nev., if PSHA is applied[14,21,23-24]. The use of PSHA also leads to the peculiar result that "the true seismic risk to life and property from code-designed buildings is very different in different parts of the country" [15,25].

The problems in the application of PSHA seem to be not only because of inadequate understanding and insufficient data on earthquakes, but also because of some technical deficiency of the method itself. The basic function of PSHA is to use spatial statistical characteristics of ground motion (ground-motion uncertainty) to extrapolate temporal characteristics of ground motion from temporal characteristics of earthquake occurrence[18-20,26], or the so-called ergodic assumption[27]. Because the occurrence of ground motion at a site must be associated with the occurrence of an earthquake, the extrapolated temporal characteristics of ground motion must be consistent with those of earthquake occurrence. PSHA fails to provide such consistency[12-13,18-20,27].

In order to explore the technical deficiency, the heart of PSHA (i. e., the basic concepts and formulations) will be re-examined first in this paper. Then an alternative approach, called KY-PSHA, will be developed to correct the deficiency. Finally, current PSHA and KY-PSHA will be compared and discussed.

1 Basic Concepts

Because PSHA was developed based on the principle of probability[1], it would be beneficial to briefly review some basic concepts of probability theory, especially the probability density function (PDF), cumulative distribution function (CDF), and exceedance probability or complementary CDF.

If a random variable X follows a normal distribution, the PDF for X is

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma_x}\exp\left(-\frac{(x-\mu_x)^2}{2\sigma_x^2}\right) \quad -\infty < x < +\infty, \tag{1}$$

where μ_x and σ_x are the mean and standard deviation, and the CDF is

$$F_X(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma_x}\exp\left(-\frac{(x-\mu_x)^2}{2\sigma_x^2}\right)dx. \tag{2}$$

The exceedance probability, $P[X \geq x]$, is

$$P[X \geq x] = 1 - F_X(x) = 1 - \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma_x}\exp\left(-\frac{(x-\mu_x)^2}{2\sigma_x^2}\right)dx. \tag{3}$$

A random variable S follows a log-normal distribution if it has

$$\ln(s) = x \quad \text{and} \quad 0 < s < +\infty, \tag{4}$$

and X follows a normal distribution. The PDF for S is

$$f_S(s) = \frac{1}{\sqrt{2\pi}\sigma_S}\exp\left(-\frac{(\ln s - \ln\mu_S)^2}{2\sigma_{\ln,S}^2}\right), \tag{5}$$

where μ_S and $\sigma_{\ln,S}$ are median (mean in log) and log standard deviation, and the CDF for S is

$$F_S(s) = \int_0^X \frac{1}{\sqrt{2\pi}\sigma_S}\exp\left(-\frac{(\ln s - \ln\mu_S)^2}{2\sigma_{\ln,S}^2}\right)d(\ln(s)). \tag{6}$$

The exceedance probability, $P[S \geq s]$, is

$$P[S \geq s] = 1 - F_S(s) = 1 - \int_0^X \frac{1}{\sqrt{2\pi}\sigma_{\ln,S}}\exp\left(-\frac{(\ln s - \ln\mu_S)^2}{2\sigma_{\ln,s}^2}\right)d(\ln(s)). \tag{7}$$

If the occurrence of $Z \geq z$ depends on the occurrence of X, the exceedance probability $P[Z \geq z]$ is equal to

$$P[Z \geq z] = \int_{-\infty}^{+\infty} P[Z \geq z \mid x] f_X(x)dx \quad -\infty \leq x \leq +\infty, \tag{8}$$

where $P[Z \geq z \mid x]$ is the conditional exceedance probability for $Z \geq z$ when x occurs.

Equation (8) is called the *total probability theorem*. Equations (1) through (8) are the probability distributions for a single random variable and simple distributions (normal and log-normal). For real applications, particularly for PSHA, we also need to know the probability distributions for complex functions. These complex functions include source-to-site configuration (line or areal source), the Gutenburg-Richter relationship (earthquake magnitude-occurrence frequency), and the ground-motion attenuation relationship[28-33].

The PDF for source-to-site distance (R),

$f_R(r)$, depends on the spatial configurations of the sources and site, and can only be determined explicitly from the specific source-to-site geometric configuration[1,34]. For example, if the source-to-site geometric configuration is as in Figure 1[1], $f_R(r)$ is equal to

$$f_R(r) = \frac{r}{50\sqrt{r^2 - 40^2}} \qquad 40 \le r \le 64. \quad (9)$$

The CDF and exceedance probability for R are

$$F_R(r) = \frac{1}{50}\sqrt{r^2 - 40^2} \qquad 40 \le r \le 64 \quad (10)$$

and

$$P[R \ge r] = 1 - F_R(r) = 1 - \frac{1}{50}\sqrt{r^2 - 40^2} \qquad 40 \le r \le 64, \quad (11)$$

respectively.

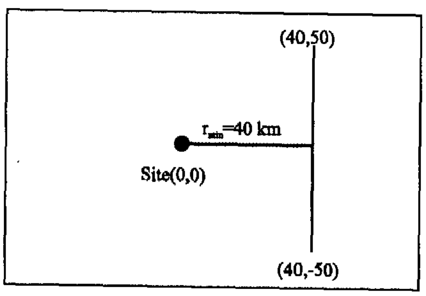


Fig. 1　A hypothetical site 40 km from a line source.

Earthquake occurrence generally follows the well-known Gutenberg-Richter relationship (function):

$$\lambda = \frac{1}{\tau} = e^{\alpha - \beta m} \qquad m_0 \le m \le m_{\max}, \quad (12)$$

where λ is the cumulative number of earthquakes with magnitude equal to or greater than m occurring yearly, τ is the recurrence interval, α and β are constants, and m_0 and $m_{\max}$ are the lower and upper bounds of earthquake magnitude, respectively. The PDF for earthquake magnitude (M) is

$$f_M(m) = \frac{\beta e^{-\beta(m-m_0)}}{1 - e^{-\beta(m_{\max}-m_0)}} \qquad m_0 \le m \le m_{\max}. \quad (13)$$

The CDF for M is

$$F_M(m) = \frac{1 - e^{-\beta(m-m_0)}}{1 - e^{-\beta(m_{\max}-m_0)}} \qquad m_0 \le m \le m_{\max}, \quad (14)$$

and the exceedance probability $P[M \ge m]$ is

$$P[M \ge m] = 1 - F_M(m) = \frac{e^{-\beta(m-m_0)} - e^{-\beta(m_{\max}-m_0)}}{1 - e^{-\beta(m_{\max}-m_0)}} \qquad m_0 \le m \le m_{\max}. \quad (15)$$

Equation (15) shows that the exceedance probability for M is a functional distribution when M follows a functional relationship (equation (12)).

In seismology, ground motion Y can be expressed as a function of M and R[28-33], a function called the ground-motion attenuation relationship:

$$\ln(Y) = f(M,R) + (E), \quad (16)$$

where E is the error (uncertainty) of $\ln(Y)$ and modeled as a normal distribution with a standard deviation, $\sigma_{\ln,Y}$. In other words, E is modeled as a log-normal distribution. For example, the attenuation relationship of Atkinson and Boore[30] is

$$\ln(Y) = c_1 + c_2(M-6) + c_3(M-6)^2 - \ln R - c_4 R + (E), \quad (17)$$

where c_1, c_2, c_3, and c_4 are empirically determined constants. As shown by this example, Y is a complex and nontrivial function of M and R. Its exceedance probability $P[Y \ge y]$ is unknown and difficult to determine. The core step of PSHA is to seek the exceedance probability[1,35-37].

2　Problem in PSHA

The purpose of PSHA is to derive the seismic hazard: a ground-motion level versus its annual probability of exceedance or return period[1,37]. As discussed earlier, the core step of PSHA is to seek the exceedance probability for the ground-motion attenuation relationship[1,35-37]. According to Cornell[1,35] and McGuire[36-37], the central part of the PSHA formulation is

$$\gamma(y) = \sum_j \nu_j P_j[Y \ge y] = \sum_j \nu_j \iint P_j[Y \ge y \mid m,r] f_{M,j}(m) f_{R,j}(r)\,dm\,dr, \quad (18)$$

where γ is the annual probability of exceedance for a ground motion $Y \ge y$, ν_j is the activity rate and equal to

$$\nu_j = e^{\alpha_j - \beta_j m_0}, \quad (19)$$

$f_{M,j}(m)$ and $f_{R,j}(r)$ are the PDF for earthquake magnitude (M) and source-to-site distance (R), respectively, and $P_j[Y \ge y \mid m, r]$ is the conditional probability that Y exceeds y (exceedance probability) at given m and r from seismic source j. From equation (18), we have *total probability theorem*:

$$P_j[Y \geq y] = \iint P_j[Y \geq y \mid m, r] f_{M,j}(m) f_{R,j}(r) \mathrm{d}m\mathrm{d}r. \tag{20}$$

Equation (20) shows that seeking the exceedance probability, $P_j[Y \geq y \mid m, r]$, conditioned at m and r, is the key in seismic hazard calculation. The conditional exceedance probability, $P_j[Y \geq y \mid m, r]$, has not been derived and is unknown. In current PSHA, however, $P_j[Y \geq y \mid m, r]$ is simply equated to[35-37]

$$P_j[Y \geq y \mid m, r] = 1 - \int_0^y \frac{1}{\sqrt{2\pi}\sigma_{\ln,y}} \exp\left(-\frac{(\ln y - \ln y_{mr})^2}{2\sigma_{\ln,y}^2}\right) \mathrm{d}(\ln(y)), \tag{21}$$

where y_{mr} is equal to

$$\ln(y_{mr}) = f(m, r). \tag{22}$$

The right side of equation (21) is the exceedance probability for a log-normal distribution (equation (7)). Here, the conditional exceedance probability, $P[Y \geq y \mid m, r]$, has been equated to the exceedance probability of the ground-motion uncertainty at given m and r. The ground-motion uncertainty is a single variable and modeled as a normal distribution with a standard deviation, $\sigma_{\ln,y}$[28-33].

Equating the conditional exceedance probability of the ground-motion attenuation relationship to the exceedance probability of the ground-motion uncertainty seems simple, but is mathematically incorrect because the two probabilities have different physical and mathematical meanings. As shown in Figure 2, the attenuation relationship describes a functional relation between ground motion and earthquake magnitude and source-to-site distance (Fig. 2(a)), whereas the uncertainty describes the probability distribution of the ground motion (a single variable) (Fig. 2(b)). Figure 2 also shows that the exceedance probability of the ground-motion uncertainty at m and r follows a log-normal distribution (the right side of equation (21)), whereas the exceedance probability of the attenuation relationship for a given y conditioned at m and r is an unknown functional distribution. Therefore, the conditional exceedance probability of the ground motion attenuation relationship (a function) is not equal to the exceedance probability of the ground-motion uncertainty (a single variable).

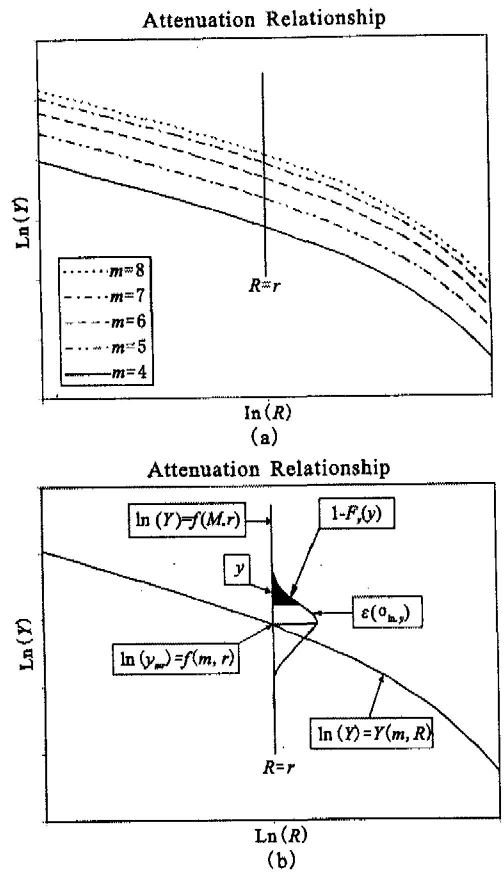


Fig. 2 Ground-motion attenuation relationship (a) and ground-motion uncertainty at a given m and r (b).

This mathematical error may have resulted from an incorrect procedure for calculating the conditional exceedance probability. In order to calculate the conditional exceedance probability at given m and r, we need to find the exceedance probability distribution (a function) first, because the ground motion is a function of M and R (equation (16) or (17)), and then evaluate the exceedance probability at given m and r. This is similar to the procedure for calculating a derivative or integration at a given point for a function in calculus: find the derivative or integration function first, and then evaluate the result at the given point. If we calculate the ground motion at given m and r first, which is incorrect, then we have

$$\ln(y) = \ln(y_{mr}) + \varepsilon. \qquad (23)$$

Here, $\ln(y_{mr})$ is a constant, and ε follows a log-normal distribution. Thus, the exceedance probability for y in equation (23) is exactly the right side of equation (21).

This mathematical error causes difficulty in understanding and applying PSHA. In current practice, the inverse of the annual probability of exceedance ($1/\gamma$), called the return period (T_P), is more often used:

$$T_P(y) = \frac{1}{\gamma(y)} = \frac{1}{\sum_j \nu_j \iint P_j[Y \geq y|m,r] f_{M,j}(m) f_{R,j}(r)\,\mathrm{d}m\mathrm{d}r}. \qquad (24)$$

If all seismic sources are characteristic, return period is

$$T_P(y) = \frac{1}{\sum_j \frac{1}{T_j}[1-\int^y \frac{1}{\sqrt{2\pi}\sigma_{\ln,c}} \exp(-\frac{(\ln y - \ln y_c)^2}{2\sigma_{\ln,c}^2})\mathrm{d}(\ln(y))]}, \qquad (25)$$

where T_j is the average recurrence interval of the characteristic earthquake and y_c and $\sigma_{\ln,c}$ are the median ground motion and standard deviation (log) for the characteristic earthquake (m_c) at the distance (r_c) from source j. For a single characteristic source, equation (25) becomes

$$T_P(y) = \frac{T}{1-\int_0^y \frac{1}{\sqrt{2\pi}\sigma_{\ln,c}} \exp(-\frac{(\ln y - \ln y_c)^2}{2\sigma_{\ln,c}^2})\mathrm{d}(\ln(y))}. \qquad (26)$$

Equations (25) and (26) demonstrate that the error in current PSHA results in extrapolation of the return period from the recurrence interval of earthquakes and the uncertainty of ground motion[18-20,26] or the so-called ergodic assumption[27].

In seismology, only a few hundreds years of instrumental and historical records and 10, 000 years of geologic records (Holocene age) on earthquakes are available. PSHA could extrapolate ground motions generated by "earthquakes" that have much longer return periods, 10^8 years or longer, however[21-23]. For example, for the proposed nuclear waste repository at Yucca Mountain, Nev., PSHA could be used to derive ground motions having a 100-million-year return period (annual probability of exceedance of 10^{-8})[21-24]. Similarly in Switzerland, PSHA could be used to derive ground motions having a 10-million-year return period (annual probability of exceedance of 10^{-7}) from a few thousand years of records[17]. According to equation (27) or (28), ground motion with a return period of 10^7 years or longer means that that ground motion has an extremely low probability (10^{-4} or less) of occurring when the associated earthquakes occur. This is the true meaning of the ground motion with a return period of 10^7 years or longer. The ground motion has been interpreted, however, as that the ground motion will occur in 10^7 years or longer[26,38]. This interpretation is incorrect and contradicts the inputs and the principle of probability.

Ground motion is a consequence of an earthquake, and occurrence of a ground motion at a site must be associated with the occurrence of an earthquake. In other words, the temporal characteristics of ground-motion occurrence must be consistent with those of earthquake occurrence. Current PSHA does not derive temporal characteristics of ground-motion occurrence that are consistent with those of earthquake occurrence. Thus, the mathematical error in current PSHA results in invalid hazard calculation.

3 Correction

As discussed earlier, seeking the exceedance probability or conditional exceedance probability for the ground-motion attenuation relationship is the core step in seismic hazard calculation. It is very complicated or even impossible to seek these probability distributions directly from the ground-motion attenuation relationship, equation (16) or (17), because it is a very complex function. In this section, I will develop an alternative formulation by utilizing known probability distributions. In order to differentiate this alternative from current PSHA, it will be called KY-PSHA.

From equation (16), Y_E (ground motion with an uncertainty $E=0$, $\pm\sigma_{\ln,Y}$, $\pm 2\sigma_{\ln,Y}$, etc.) can be equal to

$$Y_E = \ln(Y) - E = f(M,R). \qquad (27)$$

Then M can be expressed as a function of R and Y_E:

$$M = g(R, Y_E). \qquad (28)$$

Generally, ground-motion attenuation relationships are quite complicated[28-33]. At a given $R=r$, if letting

$$Y_E = f(M,r) \geq y_\varepsilon, \qquad (29)$$

we can solve for

$$M \geq m_g = g(r, y_\varepsilon) \quad m_0 \leq M \leq m_{\max}. \qquad (30)$$

The relationships between Y_E, $f(M, r)$, y_ε, M, m_g, and $g(r, y_\varepsilon)$ are shown in Figure 3. Figure 3 shows that for a given r and y_ε, equation (30) can not only be uniquely determined, but also is equivalent to equation (29). Hence, the conditional probability that ground motion Y_E exceeds y_ε at a given r is

$$P[Y_E \geq y_\varepsilon \mid r] = P[f(M,r) \geq y_\varepsilon] = P[M \geq g(r, y_\varepsilon)]. \qquad (31)$$

From equation (15), we have

$$P[M \geq g(r, y_\varepsilon)] = 1 - F_M[g(r, y_\varepsilon)] = \frac{e^{-\beta[g(r,y_\varepsilon)-m_0]} - e^{-\beta(m_{\max}-m_0)}}{1 - e^{-\beta(m_{\max}-m_0)}}. \qquad (32)$$

Therefore, we have

$$P[Y_E \geq y_\varepsilon \mid r] = \frac{e^{-\beta(g(r,y_\varepsilon)-m_0)} - e^{-\beta(m_{\max}-m_0)}}{1 - e^{-\beta(m_{\max}-m_0)}}. \qquad (33)$$

Again, equation (33) shows that the conditional exceedance probability for the ground— motion attenuation relationship at a given r is also a function of r. According to the total probability theorem (i. e., equation (8)), the probability that ground motion Y_E at a site exceeds a given y_ε for R is

$$P[Y_E \geq y_\varepsilon] = \int P[Y_E \geq y \mid r] f_R(r)\,dr = \int \frac{e^{-\beta(g(r,y_\varepsilon)-m_0)} - e^{-\beta[m_{\max}-m_0]}}{1 - e^{-\beta(m_{\max}-m_0)}} f_R(r)\,dr. \qquad (34)$$

Thus, the average annual probability that ground motion Y_E at a site exceeds a given y_ε from a source is

$$\nu P[Y_E \geq y_\varepsilon] = \nu \int \frac{e^{-\beta[g(r,y_\varepsilon)-m_0]} - e^{-\beta(m_{\max}-m_0)}}{1 - e^{-\beta(m_{\max}-m_0)}} f_R(r)\,dr. \qquad (35)$$

For all sources, the total average annual probability that ground motion Y_E at a site exceeds a given y_ε is

$$\gamma(y_\varepsilon) = \sum_j \nu_j P_j[Y_E \geq y_\varepsilon] = \sum_j \nu_j \int \frac{e^{-\beta_j[g_j(r,y_\varepsilon)-m_0]} - e^{-\beta_j(m_{\max}-m_0)}}{1 - e^{-\beta_j(m_{\max}-m_0)}} f_{R,j}(r)\,dr. \qquad (36)$$

For a single characteristic source, equation (36) becomes

$$\gamma(y_\varepsilon) = e^{\alpha-\beta m_c} = \frac{1}{T}. \qquad (37)$$

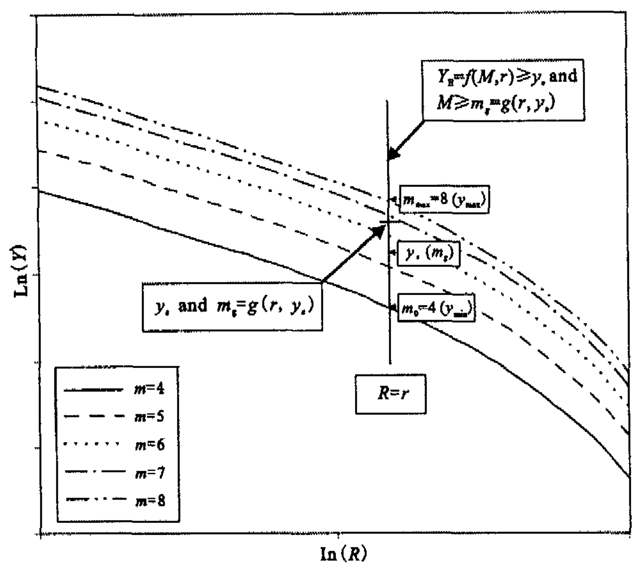


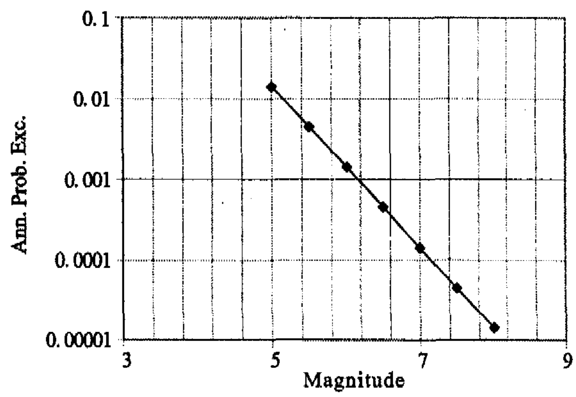
Fig. 3 Elationships between Y_E, $f(M, r)$, y_ε, M, m_g, and $g(r, y_\varepsilon)$.

Equation (37) is the new formula for seismic hazard calculation, which is different from the current formula (equation (18)). All terms in equation (36) are known and can be easily computed, especially by the numerical method. In contrast, the conditional exceedance probability in equation (18) is unknown. As shown in the equations, KY-PSHA will explicitly determine a ground motion with a specific level of uncertainty (i. e., $\varepsilon=0$, $\pm\sigma_{\ln,y}$, $\pm 2\sigma_{\ln,y}$). Also, the temporal characteristics of ground motion derived from KY-PSHA are consistent with those of an earthquake. This can be clearly seen in a single characteristic source. KY-PSHA (equation (37)) will derive a single annual probability of exceedance ($1/T$) for a single characteristic earthquake. In contrast, current PSHA (equation (26)) will derive infinite annual probabilities of exceedance for the same characteristic earthquake[12-13,26].

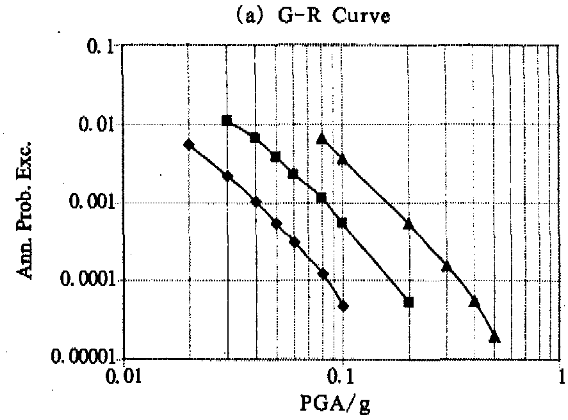
Application of KY-PSHA will be demonstrated in a simple example in which the source and site configuration are as shown in Figure 1, and $\alpha = 7.254$, $\beta = 2.303$, $m_0 = 5$, and $m_{max} = 8$. The peak ground acceleration (PGA) attenuation relationship of Campbell[29] was used. According to Campbell, PGA (Y_E) is

$$\begin{aligned} Y_E &= \ln(Y) - E \\ &= 0.0305 + 0.633M - 0.0427(8.5-M)^2 \\ &\quad - 0.7955\ln(R^2 + [0.683\exp(0.416M)]^2) \\ &\quad + (-0.00428 + 0.000483M)R \end{aligned} \tag{38}$$

for $R \leq 70$ km. According to Campbell[29], $\sigma_{\ln,Y}$ also depends on earthquake magnitude and distance. The PGA attenuation relationship, equation (38), is quite complicated, and the function $g(R, Y_E)$ cannot be solved explicitly, but can be solved implicitly (numerically). Figure 4(b) shows PGA hazard curves for the median and median $\pm\, \sigma_{\ln,y}$ at 40 km from the line source (Fig. 1). For comparison, the Gutenburg-Ritcher curve is also shown in Fig. 4(a). The annual probabilities of exceedance determined from KY-PSHA are between 0.01 and 0.000 01 (Fig. 4(b)); the annual probability of earthquakes with magnitude equal to or greater than 5.0 is about 0.01; and the annual probability of earthquakes with magnitude equal to or greater than 8.0 is about 0.000 01 (Fig. 4(a)). In other words, in terms of temporal characteristics, the outputs from KY-PSHA are consistent with the inputs. Particularly in the case of a single characteristic source, the output (the annual probabilities of exceedance) is equal to the input (recurrence rate of the characteristic earthquake). Figure 4(b) also shows that the ground-motion uncertainty is explicitly expressed in KY-PSHA. For example, for the annual probability of exceedance of 0.001 (return period of 1,000 years), we could estimate the median PGA of 0.08 g, median $-\, \sigma_{\ln,y}$ PGA of 0.04 g, and median $+\, \sigma_{\ln,y}$ PGA of 0.16 g, respectively. This is similar to the hazard estimates in hydraulic engineering, which could also give a level of uncertainty[39-40,20].



(a) G-R Curve



(b) PGA Hazard Curve

Fig. 4 Gutenburg-Ritcher (a) and PGA (b) hazard curves for a site 40 km from a line source(Square: median; diamond: median $-\, \sigma_{\ln,y}$; triangle: median $+\, \sigma_{\ln,y}$).

Although developed differently, KY-PSHA is similar to the original PSHA of Cornell[1]. In fact, KY-PSHA is identical to Cornell's method if ground-motion uncertainty is not considered (i.e., $E = 0.0$). For $E = 0.0$, if MMI (I) is equal to

$$I = b_1 + b_2 M - b_3 \ln R + (E = 0), \tag{39}$$

where b_1, b_2, and b_3 are constants[1], then

$$M = g(R, I) = \frac{I + b_3 \ln R - b_1}{b_2}. \tag{40}$$

Equation (40) is similar to equation (28) for $E = 0.0$. From equation (36), we have

$$P[I \geq i] = \int \frac{e^{-\beta[g(r,i)-m_0]} - e^{-\beta(m_{max}-m_0)}}{1 - e^{-\beta(m_{max}-m_0)}} f_R(r)\,dr. \tag{41}$$

Equation (41) is identical to equation (9) for the truncated Gutenburg-Richter relationship for $E = 0.0$. This suggests that the formulations in the original PSHA[1,35] are correct, and the error was introduced later. This was confirmed by Cornell (personal communication, 2004).

4 Conclusion

It is clear that there is a mathematical error (i. e., incorrectly equating the conditional exceedance probability of the ground-motion attenuation relationship (a function) to the exceedance probability of the ground-motion uncertainty (a variable) in the hazard calculation of current PSHA). This error may have resulted from an incorrect procedure for evaluating the conditional exceedance probability of the ground-motion attenuation relationship. The error results in using the ground-motion uncertainty (spatial distribution) to extrapolate the occurrence of ground motion (temporal distribution), or the so-called ergodic assumption[27]. The error has caused difficulty in understanding and applying PSHA. An alternative method, KY-PSHA, has been devised to correct the error. In contrast to current PSHA, KY-PSHA derives ground motions that have temporal characteristics consistent with those of the associated earthquakes. Also, ground-motion uncertainty is explicitly considered in KY-PSHA, which is similar to hazard estimates in hydraulic and wind engineering, which could also give a level of uncertainty[39-40,20].

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