桩基础在分层弹性地基场地土扭转波作用下的扭转地震反应分析

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摘要 本文对分层弹性地基中单桩基础通过特性分析,建立了合理的力学模型,按分层弹性地基土模型对桩进行扭转振动分析,给出了桩基础扭转自振特性及在扭转地震载荷与扭转振动载荷作用下的强迫反应解析解,文中的解析公式为分层弹性地基中的桩基础扭转地震反应分析提供了一种新的解析方法。

主题词:扭转波 地震反应分析 桩 地基

1 前言

在某些建筑结构中,桩是最基本的构件[1],本文着重分析桩在扭转地震载荷作用下的动力反应问题。首先将桩头部各种动力简化成扭转振动载荷,此外,桩还受到自身材料阻尼扭矩与惯性扭矩,地基抗扭力矩与非线性辐射阻尼扭矩,扭转地震力与地基扭转波动等的影响。在研究中近似地把分层弹性地基与基岩或相对硬层看成水平层状介质,并且沿水平方向每层本身为各向同性匀质体,对于分层弹性地基可取为单位截面积的竖向土柱来分析其波动问题[2,8]。在考虑上述问题的基础上建立了基本扭转振动方程,继而求得了解析解。

2 地震反应的基本扭转振动方程

由于场地土随着深度的不同而分层变化,其分析的典型图见图1—3。在图1—3中$m_x^{(t)}$为桩头绕x轴的动扭矩,$\varphi_g^{(t)}$为地震扭转角位移,$\psi_i^{(x,t)}$为i层地基对基岩或相对硬层运动的相对角位移(或称自身扭转振动角位移),X_i为i段桩和i层地基的座标,M_i与M_n和M_{ci}与M_{cn}及M_{pi}与M_{pn}为i与n段桩的截面内扭矩和材料阻尼扭矩及惯性扭矩,m_{ki}为地基分布摩擦扭力矩,m_{fi}为地基对桩产生的分布辐射阻尼扭矩,M_{Ti}与M_{ti}及M_{tci}为i层地基的内扭矩与惯性扭矩及阻尼扭矩,r_{1i}为i段轴心到单位截面积土柱表面的半径,r_{2i}为i段轴心到桩表面的半径,Δ_{ti}为i段单位截面积土柱的厚度。它们分别为:

$$M_i=G_iJ_i\frac{\partial\varphi_i}{\partial x_i},\quad M_n=G_nJ_n\frac{\partial\varphi_n}{\partial x_n},\quad M_{ci}=C_i\frac{\partial\varphi_i}{\partial t}dx_i,\quad M_{cn}=C_n\frac{\partial\varphi_n}{\partial t}dx_n,$$

$$M_{pi}=I_{pi}\frac{\partial^2\bar{\varphi}_i}{\partial t^2},\quad M_{pn}=I_{pn}\frac{\partial^2\bar{\varphi}_n}{\partial t^2},\quad m_{ki}=K_{\varphi i}(\bar{\varphi}_i-\bar{\psi}_i),\quad m_{fi}=C_{fi}\left(\frac{\partial\bar{\varphi}_i}{\partial t}-\frac{\partial\bar{\psi}_i}{\partial t}\right),$$

$$M_{Ti}=G_{Ti}J_{Ti}\frac{\partial\psi_i}{\partial x_i},\quad M_{ti}=I_{ti}\frac{\partial^2\psi_i}{\partial t^2},\quad M_{tci}=C_{ti}\frac{\partial\psi_i}{\partial t}dx_i$$

式中G_i、G_n，J_i、J_n，C_i、C_n以及I_{pi}、I_{pn}分别为桩的第i与n段的剪切模量、截面抗扭常数、材料阻尼系数及转动惯量；G_{Ti}、J_{Ti}、C_{ti}及I_{ti}分别为第i层地基的剪切模量、截面抗扭常数、材料阻尼系数[3]及转动惯量；$K_{\varphi i}$与C_{fi}为第i层地基的摩擦力矩系数与非线性辐射阻尼系数；$\bar{\varphi}_i^{(x_i,t)}$、$\bar{\varphi}_n^{(x_n,t)}$和$\bar{\psi}_i^{(x_i,t)}$分别为第i与n段桩和第i层地基对基岩或相对硬层运动的绝对角位移；$\varphi_i^{(x_i,t)}$、$\varphi_n^{(x_n,t)}$和$\psi_i^{(x_i,t)}$为第i与n段桩和第i层地基对基岩和相对硬层运动的相对角位移(或称为自身振动角位移)。

$$I_{pi}=\rho_{pi}J_{pi}^{(x_i)}dx_i,\qquad I_{pn}=\rho_{pn}J_{pn}^{(x_n)}dx_n,\qquad I_{ti}=\rho_{ti}J_{ti},$$
$$\bar{\varphi}_i^{(x_i,t)}=\varphi_i^{(x_i,t)}+\varphi_g^{(t)},\quad \bar{\varphi}_n^{(x_n,t)}=\varphi_n^{(x_n,t)}+\varphi_g^{(t)},\quad \bar{\psi}_i^{(x_i,t)}=\psi_i^{(x_i,t)}+\varphi_g^{(t)}$$

式中ρ_{pi}、ρ_{pn}和ρ_{ti}以及J_{pi}、J_{pn}和J_{ti}为第i与n段桩和第i层土的质量密度以及极惯性矩。第i段单位截面积土柱的截面抗扭常数为

$$J_{Ti}=\frac{1}{4}\pi\bar{D}_{ti}^3\Delta_{ti}=2\pi\bar{r}_{ti}^3\Delta_{ti}=2\pi(r_{zi}+\frac{1}{2}\Delta_{ti})^3\Delta_{ti}$$
$$=\frac{1}{4}\pi r_{zi}^4(\sqrt{1+\frac{1}{\pi r_{zi}^2}}+1)^3(\sqrt{1+\frac{1}{\pi r_{zi}^2}}-1)\qquad(1)$$

式中$\bar{r}_{ti}$为轴心到土柱的平均半径，即

$$\bar{r}_{ti}=r_{zi}+\frac{1}{2}\Delta_{ti}=\frac{1}{2}\bar{D}_{ti}\qquad(2)$$

单位截面积土柱的横截面积

$$F_t=\pi r_{ti}^2-\pi r_{zi}^2=\pi(r_{ti}-r_{zi})(r_{ti}+r_{zi})$$
$$=\pi\Delta_{ti}(2r_{ti}+\Delta_{ti})=1\qquad(3)$$

因而有关于Δ_{ti}的一元二次方程为：

$$\Delta_{ti}^2+2r_{zi}\Delta_{ti}-\frac{1}{\pi}=0\qquad(4)$$

从而解得单位截面积土柱的厚度为：

$$\Delta_{ti}=[(1+\frac{1}{\pi r_{zi}^2})^{\frac{1}{2}}-1]r_{zi}\qquad(5)$$

从轴心到单位截面积土柱外面的半径为

$$r_{ti}=r_{zi}+\Delta_{ti}=(1+\frac{1}{\pi r_{zi}^2})^{\frac{1}{2}}r_{zi}\qquad(6)$$

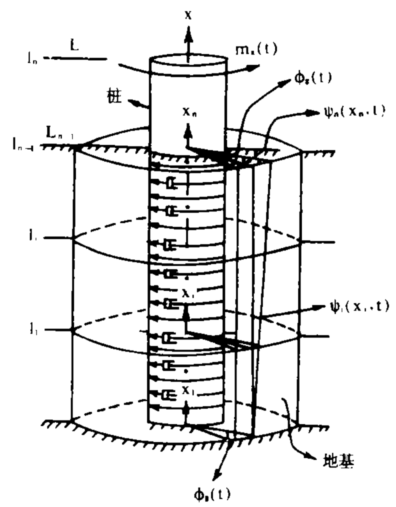

图1 桩体受力分析图

Fig. 1 The analysis of forces borne by a pile.

对于圆形截面有：

$$J_i=J_{pi},\quad J_n=J_{pn},\quad J_{Ti}=J_{ti}$$

按图2与图3根据平衡条件可分别建立桩与弹性地基[8]的基本地震扭转反应振动方程式为(式中δ为单位脉冲函数)：

$$G_iJ_i\frac{\partial^2\varphi_i}{\partial x_i^2}-(C_i+C_{fi})\frac{\partial\varphi_i}{\partial t}-K_{\varphi i}\varphi_i-\rho_{pi}J_{pi}\frac{\partial^2\varphi_i}{\partial t^2}=\rho_{pi}J_{pi}\frac{d^2\varphi_g^{(t)}}{dt^2}-K_{\varphi i}\psi_i-C_{fi}\frac{\partial\psi_i}{\partial t}\qquad(7)$$

$$G_nJ_n\frac{\partial^2\varphi_n}{\partial x_n^2}-C_n\frac{\partial\varphi_n}{\partial t}-\rho_{pn}J_{pn}\frac{\partial^2\varphi_n}{\partial t^2}=\rho_{pn}J_{pn}\frac{d^2\varphi_g^{(t)}}{dt^2}-m_x^{(t)}\delta(X-L)\qquad(8)$$

$$G_{Ti}J_{Ti}\frac{\partial^2\psi_i}{\partial x_i^2}-C_{ti}\frac{\partial\psi_i}{\partial t}-\rho_{ti}J_{ti}\frac{\partial^2\psi_i}{\partial t^2}=\rho_{ti}J_{ti}\frac{d^2\varphi_g^{(t)}}{dt^2}\qquad(9)$$

3 基本方程的自由扭转振动解

对于自由扭转振动问题，7、8、9三式可设解为：

$$\varphi_i^{(x_i,t)}=\Phi_i^{(x_i)}T_{zi}^{(t)},\qquad \varphi_n^{(x_n,t)}=\Phi_n^{(x_n)}T_{zn}^{(t)},\qquad \psi_i^{(x_i,t)}=\Psi_i^{(x_i)}T_{ti}^{(t)}$$

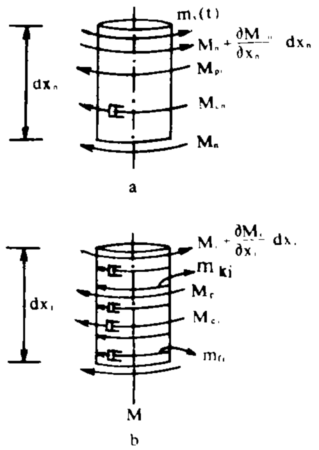

图2　桩体第 i 段与第 n 段微元体

Fig. 2　Microelements in segments i and n of the pile.

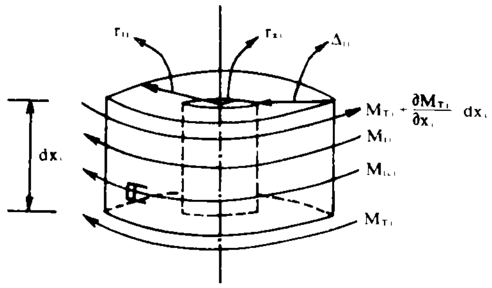

图3　桩体第 i 段单位截面积土柱微元体

Fig. 3　Microelements of soil pillar on unit area of the segment i.

式中$T_{zi}^{(t)}$、$T_{zn}^{(t)}$和$T_{ti}^{(t)}$为自由扭转振动时间函数,$\Phi_i^{(x_i)}$、$\Phi_n^{(x_n)}$和$\Psi_i^{(x_i)}$为振型函数,将上述解分别代入7、8、9三式并分离变量有:

$$-\left(\frac{G_iJ_i}{\rho_{pi}J_{pi}}\Phi''^{(x_i)}_i-\frac{K_\varphi}{\rho_{pi}J_{pi}}\Phi_i^{(x_i)}\right)/\Phi_i^{(x_i)}=-\left(\ddot{T}^{(t)}_{z,i}+\frac{C_i+C_{fi}}{\rho_{pi}J_{pi}}\dot{T}^{(t)}_{zi}\right)/T^{(t)}_{z,i}=\omega^2 \tag{10}$$

$$-\frac{G_nJ_n}{\rho_{pn}J_{pn}}\Phi''^{(x_n)}_n/\Phi_n^{(x_n)}=-\left(\ddot{T}^{(t)}_{zn}+\frac{C_n}{\rho_{pn}J_{pn}}\dot{T}^{(t)}_{zn}\right)/T^{(t)}_{z,n}=\omega^2 \tag{11}$$

$$-\frac{G_{Ti}J_{Ti}}{\rho_{ti}J_{ti}}\Psi''^{(x_i)}_i/\Psi_i^{(x_i)}=-\left(\ddot{T}^{(t)}_{ti}+\frac{C_{ti}}{\rho_{ti}J_{ti}}\dot{T}^{(t)}_{ti}\right)/T^{(t)}_{ti}=\omega_{ti}^2 \tag{12}$$

式中 ω 与ω_{ti}为桩与土的自振圆频率,上述3式可分别写为:

$$\Phi''^{(x_i)}_i+\xi_i^2\Phi_i^{(x_i)}=0,\qquad \ddot{T}^{(t)}_{zi}+2\mu_i\omega\dot{T}^{(t)}_{zi}+\omega^2T^{(t)}_{zi}=0 \tag{13}$$

$$\Phi''^{(x_n)}_n+\alpha_n^2\Phi_n^{(x_n)}=0,\qquad \ddot{T}^{(t)}_{zn}+2\mu_n\omega\dot{T}^{(t)}_{zn}+\omega^2T^{(t)}_{zn}=0 \tag{14}$$

$$\Psi''^{(x_i)}_i+\theta_i^2\Psi_i^{(x_i)}=0,\qquad \ddot{T}^{(t)}_{ti}+2\mu_{ti}\omega_{ti}\dot{T}^{(t)}_{ti}+\omega_{ti}^2T^{(t)}_{ti}=0 \tag{15}$$

其中

$$\xi_i^2=\frac{\omega^2}{\zeta_i^2}-\frac{K_\varphi}{G_iJ_i},\qquad \zeta_i=\sqrt{\frac{G_iJ_i}{\rho_{pi}J_{pi}}},\qquad \mu_i=\frac{C_i+C_{fi}}{2\omega\rho_{pi}J_{pi}},\qquad \gamma_i=\sqrt{1-\mu_i^2}$$

$$\alpha_n^2=\frac{\omega^2}{\zeta_n^2},\qquad \zeta_n=\sqrt{\frac{G_nJ_n}{\rho_{pn}J_{pn}}},\qquad \mu_n=\frac{C_n}{2\omega\rho_{pn}J_{pn}},\qquad \gamma_n=\sqrt{1-\mu_n^2}$$

$$\theta_i=\frac{\omega_{ti}}{\eta_i},\qquad \eta_i=\sqrt{\frac{G_{Ti}J_{Ti}}{\rho_{ti}J_{ti}}},\qquad \mu_{ti}=\frac{C_{ti}}{2\omega_{ti}\rho_{ti}J_{ti}},\qquad \gamma_{ti}=\sqrt{1-\mu_{ti}^2}$$

得(13)、(14)、(15)式的解分别为:

$$\Phi_i^{(x_i)}=C_{1,i}\sin\xi_ix_i+C_{2,i}\cos\xi_ix_i,\qquad T^{(t)}_{zi}=exp(-\mu_i\omega t)(a_i\sin\gamma_i\omega t+b_i\cos\gamma_i\omega t) \tag{16}$$

$$\Phi_n^{(x_n)}=C_{1,n}\sin\alpha_nx_n+C_{2,n}\cos\alpha_nx_n,\qquad T^{(t)}_{zn}=exp(-\mu_n\omega t)(a_n\sin\gamma_n\omega t+b_n\cos\gamma_n\omega t) \tag{17}$$

$$\Psi_i^{(x_i)}=\overline{C}_{1,i}\sin\theta_i x_i+\overline{C}_{2,i}\cos\theta_i x_i, \qquad T_{ii}^{(t)}=exp(-\mu_{ii}\omega_{ii}t)(a_{ii}\sin\gamma_{ii}\omega_{ii}t+b_{ii}\cos\gamma_{ii}\omega_{ii}t) \tag{18}$$

式中系数 a_i、b_i、a_n、b_n、a_{ii}、b_{ii}由初始条件确定。

4 边界条件与变形连续条件

在 $x_1=0$ 处，认为是基岩或相对硬层，做为固定端，在中间以分层土的连接处做为变形连续条件，L_{n-1}处为地基自由表面，L 处为桩自由端。

4.1 桩基础的边界条件与变形连续条件

$$\left.\begin{array}{lll}(1)\quad x_1=0: & \Phi_1^{(0)}=0 & \\ (2)\quad x_{i-1}=l_{i-1},x_i=0: & \Phi_{i-1}^{(l_{i-1})}=\Phi_i^{(0)}, & \Phi_{i-1}'^{(l_{i-1})}=\Phi_i'^{(0)} \\ (3)\quad x_n=l_n: & \Phi_n'^{(l_n)}=0 & \end{array}\right\} \tag{19}$$

4.2 弹性地基的边界条件与变形连续条件

$$\left.\begin{array}{lll}(1)\quad x_1=0: & \Psi_1^{(0)}=0 & \\ (2)\quad x_{i-1}=l_{i-1},x_i=0: & \Psi_{i-1}^{(l_{i-1})}=\Psi_i^{(0)}, & \Psi_{i-1}'^{(l_{i-1})}=\Psi_i'^{(0)} \\ (3)\quad x_{n-1}=l_{n-1}: & \Psi_{n-1}'^{(l_{n-1})}=0 & \end{array}\right\} \tag{20}$$

5 频率方程

5.1 对于桩基础

当 16、17、两式中前一式满足(19)式后，考虑到对于动力问题各种振型都将激发，因而有第 m 阶频率矩阵式为：

$$\begin{bmatrix}[A_{1,1,m}] & & & & 0\\ & \ddots & & & \\ & & [A_{i-1,i,m}] & & \\ & & & [A_{i,i+1,m}] & \\ 0 & & & \ddots & [A_{n,n,m}]\end{bmatrix}\begin{Bmatrix}\{C_{1,1,m}\}\\ \vdots \\ \{C_{i-1,i,m}\}\\ \{C_{i,i+1,m}\}\\ \vdots \\ \{C_{n,n,m}\}\end{Bmatrix}=[A_m]\{C_m\}=0 \quad (2\leqslant i\leqslant n-1) \tag{21}$$

矩阵中各元素为：

$$[A_{1,1,m}]=[A_{1,1,m}^0 \quad A_{2,1,m}^0]$$

$$[A_{i-1,i,m}]=\begin{bmatrix}A_{1,1,m}^{i-1} & A_{2,1,m}^{i-1} & A_{3,1,m}^{i} & A_{4,1,m}^{i}\\ A_{1,2,m}^{i-1} & A_{2,2,m}^{i-1} & A_{3,2,m}^{i} & A_{4,2,m}^{i}\end{bmatrix}$$

$$[A_{i,i+1,m}]=\begin{bmatrix}A_{1,1,m}^{i} & A_{2,1,m}^{i} & A_{3,1,m}^{i+1} & A_{4,1,m}^{i+1}\\ A_{1,2,m}^{i} & A_{2,2,m}^{i} & A_{3,2,m}^{i+1} & A_{4,2,m}^{i+1}\end{bmatrix}$$

$$[A_{n,n,m}]=[A_{1,1,m}^n \ A_{2,1,m}^n]$$

$$\{C_{1,1,m}\}=[C_{1,1,m} \quad C_{2,1,m}]^T$$

$$\{C_{i-1,i,m}\}=[C_{1,i-1,m} \quad C_{2,i-1,m} \quad C_{1,i,m} \quad C_{2,i,m}]^T$$

$$\{C_{i,i+1,m}\}=[C_{1,i,m} \quad C_{2,i,m} \quad C_{1,i+1,m} \quad C_{2,i+1,m}]^T$$

$$\{C_{n,n,m}\}=[C_{1,n,m} \quad C_{2,n,m}]^T$$

$A_{1,1,m}^0=0$, $\qquad A_{2,1,m}^0=1$

$A_{1,1,m}^{i-1}=\sin\xi_{i-1,m}l_{i-1}$, $\qquad A_{2,1,m}^{i-1}=\cos\xi_{i-1,m}l_{i-1}$, $\qquad A_{3,1,m}^{i}=0$, $\qquad A_{4,1,m}^{i}=-1$

$A_{1,2,m}^{i-1}=\xi_{i-1,m}\cos\xi_{i-1,m}l_{i-1}$, $A_{2,2,m}^{i-1}=-\xi_{i-1,m}\sin\xi_{i-1,m}l_{i-1}$, $A_{3,2,m}^{i}=-\xi_{i,m}$, $A_{4,2,m}^{i}=0$

$A_{1,1,m}^{i}=\sin\xi_{i,m}l_i$, $A_{2,1,m}^{i}=\cos\xi_{i,m}l_i$, $A_{3,1,m}^{i+1}=0$, $A_{4,1,m}^{i+1}=-1$

$A_{1,2,m}^{i}=\xi_{i,m}\cos\xi_{i,m}l_i$, $A_{2,2,m}^{i}=-\xi_{i,m}\sin\xi_{i,m}l_i$, $A_{3,2,m}^{i+1}=-\xi_{i+1,m}$, $A_{4,2,m}^{i+1}=0$

$A_{1,1,m}^{n}=\cos\alpha_{n,m}l_n$, $A_{2,1,m}^{n}=-\sin\alpha_{n,m}l_n$

由(21)式知$\{C_m\}\neq0$,要使(21)式有非零解的必要条件是:

$$Det[A_m]=0 \tag{22}$$

展开后,便可解得自由振动频率方程,进而可解得一系列桩自由扭转振动圆频率ω_m(m=1,2……)。

5.2　对于分层弹性地基

当(18)式中前一式满足(20)式后,考虑到对于动力问题各种振型都将激发,因而有第m阶频率矩阵式为:

$$\begin{bmatrix} [B_{1,1,m}] & & & & 0 \\ & \ddots & & & \\ & & [B_{i-1,i,m}] & & \\ & & & [B_{i,i+1,m}] & \\ & & & & \ddots \\ 0 & & & & [B_{n-1,n-1,m}] \end{bmatrix} \begin{Bmatrix} \{D_{1,1,m}\} \\ \vdots \\ \{D_{i-1,i,m}\} \\ \{D_{i,i+1,m}\} \\ \vdots \\ \{D_{n-1,n-1,m}\} \end{Bmatrix} = [B_m]\{D_m\}=0 \quad (2\leqslant i\leqslant n-2) \tag{23}$$

矩阵中各元素为

$$[B_{1,1,m}]=[B_{1,1,m}^{0}\quad B_{2,1,m}^{0}]$$

$$[B_{i-1,i,m}]=\begin{bmatrix} B_{1,1,m}^{i-1} & B_{2,1,m}^{i-1} & B_{3,1,m}^{i} & B_{4,1,m}^{i} \\ B_{1,2,m}^{i-1} & B_{2,2,m}^{i-1} & B_{3,2,m}^{i} & B_{4,2,m}^{i} \end{bmatrix}$$

$$[B_{i,i+1,m}]=\begin{bmatrix} B_{1,1,m}^{i} & B_{2,1,m}^{i} & B_{3,1,m}^{i+1} & B_{4,1,m}^{i+1} \\ B_{1,2,m}^{i} & B_{2,2,m}^{i} & B_{3,2,m}^{i+1} & B_{4,2,m}^{i+1} \end{bmatrix}$$

$$[B_{n-1,n-1,m}]=[B_{1,1,m}^{n-1}\quad B_{2,1,m}^{n-1}]$$

$$\{D_{1,1,m}\}=[\overline{C}_{1,1,m}\quad \overline{C}_{2,1,m}]^T$$

$$\{D_{i-1,i,m}\}=[\overline{C}_{1,i-1,m}\quad \overline{C}_{2,i-1,m}\quad \overline{C}_{1,i,m}\quad \overline{C}_{2,i,m}]^T$$

$$\{D_{i,i+1,m}\}=[\overline{C}_{1,i,m}\quad \overline{C}_{2,i,m}\quad \overline{C}_{1,i+1,m}\quad \overline{C}_{2,i+1,m}]^T$$

$$\{D_{n-1,n-1,m}\}=[\overline{C}_{1,n-1,m}\quad \overline{C}_{2,n-1,m}]^T$$

$B_{1,1,m}^{0}=0$, $B_{2,1,m}^{0}=1$

$B_{1,1,m}^{i-1}=\sin\theta_{i-1,m}l_{i-1}$, $B_{2,1,m}^{i-1}=\cos\theta_{i-1,m}l_{i-1}$, $B_{3,1,m}^{i}=0$, $B_{4,1,m}^{i}=-1$

$B_{1,2,m}^{i-1}=\theta_{i-1,m}\cos\theta_{i-1,m}l_{i-1}$, $B_{2,2,m}^{i-1}=-\theta_{i-1,m}\sin\theta_{i-1,m}l_{i-1}$, $B_{3,2,m}^{i}=-\theta_{i,m}$, $B_{4,2,m}^{i}=0$

$B_{1,1,m}^{i}=\sin\theta_{i,m}l_i$, $B_{2,1,m}^{i}=\cos\theta_{i,m}l_i$, $B_{3,1,m}^{i+1}=0$, $B_{4,1,m}^{i+1}=-1$

$B_{1,2,m}^{i}=\theta_{i,m}\cos\theta_{i,m}l_i$, $B_{2,2,m}^{i}=-\theta_{i,m}\sin\theta_{i,m}l_i$, $B_{3,2,m}^{i+1}=-\theta_{i+1,m}$, $B_{4,2,m}^{i+1}=0$

$B_{1,1,m}^{n-1}=\cos\theta_{n-1,m}l_{n-1}$, $B_{2,1,m}^{n-1}=-\sin\theta_{n-1,m}l_{n-1}$

由(23)式知$\{D_m^{*}\}\neq0$,要使(23)式有非零解的必要条件是:

$$Det[B_m]=0 \tag{24}$$

展开后,便可解得地基的自振频率方程,进而可解得一系列地基自由扭转振动圆频率ω_{im}(m=1,2……)。

6 振型系数与振型函数及时间函数解

由(22)、(24)两式解得 ω_m 与 ω_{tim} 后再代回(21)、(23)两式便可解得振型系数，继而可解得振型函数与时间函数为：

$$\delta_{j,i,m}=C_{j,i,m},\quad \delta_{j,n,m}=C_{j,n,m},\quad d_{j,i,m}=\overline{C}_{j,i,m}\quad (j=1,2) \tag{25}$$

$$\Phi_{i,m}^{(x_i)}=\delta_{1,i,m}\sin\xi_{i,m}x_i+\delta_{2,i,m}\cos\xi_{i,m}x_i \tag{26}$$

$$T_{z,i,m}^{(t)}=exp(-\mu_{i,m}\omega_m t)(a_{i,m}\sin\gamma_{i,m}\omega_m t+b_{i,m}\cos\gamma_{i,m}\omega_m t) \tag{27}$$

$$\Phi_{n,m}^{(x_n)}=\delta_{1,n,m}\sin\alpha_{n,m}x_n+\delta_{2,n,m}\cos\alpha_{n,m}x_n \tag{28}$$

$$T_{z,n,m}^{(t)}=exp(-\mu_{n,m}\omega_m t)(a_{n,m}\sin\gamma_{n,m}\omega_m t+b_{n,m}\cos\gamma_{n,m}\omega_m t) \tag{29}$$

$$\Psi_{i,m}^{(x_i)}=d_{1,i,m}\sin\theta_{i,m}x_i+d_{2,i,m}\cos\theta_{i,m}x_i \tag{30}$$

$$T_{t,i,m}^{(t)}=exp(-\mu_{t,i,m}\omega_{t,i,m}t)(a_{t,i,m}\sin\gamma_{t,i,m}\omega_{t,i,m}t+b_{t,i,m}\cos\gamma_{t,i,m}\omega_{t,i,m}t) \tag{31}$$

7 强迫扭转地震反应解

对于强迫扭转振动的地震反应，可设解为：

$$\varphi^{(x,t)}=\sum_{m=1}^{\infty}\varphi_m^{(x,t)}=\sum_{m=1}^{\infty}\Phi_m^{(x)}T_{q,m}^{(t)},\quad \Psi^{(x,t)}=\sum_{m=1}^{\infty}\Psi_m^{(x,t)}=\sum_{m=1}^{\infty}\Psi_m^{(x)}T_{t,m}^{(t)} \tag{32}$$

式中 $T_{q,m}^{(t)}$ 为强迫反应时间函数，桩整体在 x 座标下承受扭转振动的地震反应，其方程应为：

$$G^{(x)}J^{(x)}\frac{\partial^2\varphi}{\partial x^2}-(c^{(x)}+c_f^{(x)})\frac{\partial\varphi}{\partial t}-K_\varphi^{(x)}\varphi-\rho_p^{(x)}J_p^{(x)}\frac{\partial^2\varphi}{\partial t^2}=$$

$$=\rho_p^{(x)}J_p^{(x)}\frac{d^2\varphi_g^{(t)}}{dt^2}-K_\varphi^{(x)}\psi-c_f^{(x)}\frac{\partial\psi}{\partial t}-m_z^{(t)}\delta(X-L) \tag{33}$$

将(32)式代入(33)式，由(10)—(12)式可知

$$G^{(x)}J^{(x)}\Phi_m''^{(x)}+\omega_m^2\rho_p^{(x)}J_p^{(x)}\Phi_m^{(x)}=0$$

代入后，对式两边乘以 $\Phi_k^{(x)}$ 并沿桩全长积分有：

$$\int_0^L\sum_{m=1}^{\infty}[\ddot{T}_{q,m}^{(t)}+2\mu_m\omega_m\dot{T}_{q,m}^{(t)}+\omega_m^2T_{q,m}^{(t)}]\rho_p^{(x)}J_p^{(x)}\Phi_m^{(x)}\Phi_k^{(x)}dx=$$

$$=\int_0^L[\rho_p^{(x)}J_p^{(x)}\frac{d^2\varphi_g^{(t)}}{dt^2}-K_\varphi^{(x)}\psi-c_f^{(x)}\frac{\partial\psi}{\partial t}-m_z^{(t)}\delta(X-L)]\Phi_k^{(x)}dx \tag{34}$$

利用结构分析非经典理论一维正交性关系[4-7]

$$\int_0^L\rho_p^{(x)}J_p^{(x)}\Phi_m^{(x)}\Phi_k^{(x)}dx\begin{cases}\neq 0 & \text{当 } m=k \text{ 时}\\ =0 & m\neq k\end{cases}$$

便得强迫扭转反应的时间函数微分方程为：

$$\ddot{T}_{q,i,m}^{(t)}+2\mu_{i,m}\omega_m\dot{T}_{q,i,m}^{(t)}+\omega_m^2T_{q,i,m}^{(t)}=M_{i,m}^{(t)}/I_m \tag{35}$$

$$\ddot{T}_{q,n,m}^{(t)}+2\mu_{n,m}\omega_m\dot{T}_{q,n,m}^{(t)}+\omega_m^2T_{q,n,m}^{(t)}=M_{n,m}^{(t)}/I_m \tag{36}$$

式中广义转动惯量 I_m 和广义扭矩 $M_{i,m}^{(t)}$ 与 $M_{n,m}^{(t)}$ 分别为

$$I_m=\int_0^L\rho_p^{(x)}J_p^{(x)}\Phi_m^{2(x)}dx=\sum_{i=1}^{n-1}\int_0^{l_i}\rho_{pi}J_{pi}\Phi_{i,m}^{2(x_i)}dx_i+\int_0^{l_n}\rho_{pn}J_{pn}\Phi_{n,m}^{2(x_n)}dx_n \tag{37}$$

$$M_{i,m}^{(t)}=\int_0^{L_{n-1}}(\rho_p^{(x)}J_p^{(x)}\frac{d^2\varphi_g}{dt^2}-K_\varphi^{(x)}\psi-C_f^{(x)}\frac{\partial\psi}{\partial t})\Phi_m^{(x)}dx$$

$$= \sum_{i=1}^{n-1} \int_0^{l_i} (\rho_{pi}^{(x_i)} J_{pi}^{(x_i)} \frac{d^2 q_i}{d t^2} - K_{\varphi i}\psi_i - C_{f i}\frac{\partial \psi_i}{\partial t}) \Phi_{i,m}^{(x_i)} d x_i$$

$$= I_{i,m}\frac{d^2 q_i}{d t^2} - \sum_{j=1}^{n-1} U_{i,j,m}^{(l_j)} exp(-\mu_{t,j,m}\omega_{t,j,m} t)(A_{t,j,m} sin\gamma_{t,j,m}\omega_{t,j,m} t + B_{t,j,m} cos\gamma_{t,j,m}\omega_{t,j,m} t) \quad (38)$$

$$M_{n,m}^{(t)} = \int_0^{l_n} [\rho_{pn}^{(x_n)} J_{pn}^{(x_n)} \frac{d^2 q_i}{d t^2} - m_z^{(t)} \delta(x_n - l_n)] \Phi_{n,m}^{(x_n)} d x_n = I_{n,m}\frac{d^2 q_i}{d t^2} - \Phi_{n,m}^{(l_n)} m_z^{(t)} \quad (39)$$

$$\left.\begin{aligned}
&U_{i,j,m}^{(l_j)} = \int_0^{l_i} \Phi_{i,m}^{(x_i)} \Psi_{j,m}^{(x_i)} d x_i = \frac{1}{2}[(\delta_{2,i,m} d_{2,j,m} - \delta_{1,i,m} d_{1,j,m}) \frac{sin(\xi_{i,m} l_i + \theta_{j,m} l_j)}{\xi_{i,m} + \theta_{j,m}} + \\
&+ (\delta_{2,i,m} d_{2,j,m} + \delta_{1,i,m} d_{1,j,m}) \frac{sin(\xi_{i,m} l_i - \theta_{j,m} l_j)}{\xi_{i,m} + \theta_{j,m}} + (\delta_{1,i,m} d_{2,j,m} + \delta_{2,i,m} d_{1,j,m}) \cdot \\
&\cdot \frac{1 - cos(\xi_{i,m} l_i + \theta_{j,m} l_j)}{\xi_{i,m} + \theta_{j,m}} + (\delta_{1,i,m} d_{2,j,m} - \delta_{2,i,m} d_{1,j,m}) \frac{1 - cos(\xi_{i,m} l_i - \theta_{j,m} l_j)}{\xi_{i,m} - \theta_{j,m}}] \\
&I_{i,m} = \sum_{i=1}^{n-1} \int_0^{l_i} \rho_{pi}^{(x_i)} J_{p,i}^{(x_i)} \Phi_{i,m}^{(x_i)} d x_i = \sum_{i=1}^{n-1} \rho_{pi} J_{pi} \xi_{i,m}^{-1} [\delta_{1,i,m}(1 - cos\xi_{i,m} l_i) + \delta_{2,i,m} sin\xi_{i,m} l_i] \\
&I_{n,m} = \int_0^{l_n} \rho_{pn}^{(x_n)} J_{p,n}^{(x_n)} \Phi_{n,m}^{(x_n)} d x_n = \rho_{p,n} J_{p,n} \alpha_{n,m}^{-1} [\delta_{1,n,m}(1 - cos\alpha_{n,m} l_n) + \delta_{2,n,m} sin\alpha_{n,m} l_n] \\
&A_{t,j,m} = a_{t,j,m}(K_{\varphi,j} - c_{f,j}\mu_{t,j,m}\omega_{t,j,m}) - b_{t,j,m} c_{f,j}\gamma_{t,j,m}\omega_{t,j,m} \\
&B_{t,j,m} = a_{t,j,m} c_{f,j}\gamma_{t,j,m}\omega_{t,j,m} + b_{t,j,m}(K_{\varphi,j} - c_{f,j}\mu_{t,j,m}\omega_{t,j,m})
\end{aligned}\right\} \quad (40)$$

(35)、(36)两式由杜哈梅积分可得强迫反应时间函数解为：

$$\begin{aligned}
T_{q,i,m}^{(t)} &= \frac{1}{\gamma_{i,m}\omega_m I_m} \int_0^t M_{i,m}^{(\tau)} exp(-\mu_{i,m}\omega_m(t-\tau)) sin\gamma_{i,m}\omega_m(t-\tau) d\tau \\
&= -\frac{1}{\gamma_{i,m}\omega_m I_m} exp(-\mu_{i,m}\omega_m t)(a_{i,j,m} sin\gamma_{i,m}\omega_m t + b_{i,j,m} cos\gamma_{i,m}\omega_m t) - \\
&- \frac{1}{\gamma_{i,m}\omega_m I_m} \sum_{j=1}^{n-1} U_{i,j,m}^{(l_j)} exp(-\mu_{t,j,m}\omega_{t,j,m} t)(c_{i,j,m} sin\gamma_{t,j,m}\omega_{t,j,m} t + d_{i,j,m} cos\gamma_{t,j,m}\omega_{t,j,m} t) + \\
&+ \frac{I_{i,m}}{\gamma_{i,m}\omega_m I_m} R_{i,m}^{(t)}
\end{aligned} \quad (41)$$

$$\begin{aligned}
T_{q,n,m}^{(t)} &= \frac{1}{\gamma_{n,m}\omega_m I_m} \int_0^t M_{n,m}^{(\tau)} exp(-\mu_{n,m}\omega_m(t-\tau)) sin\gamma_{n,m}\omega_m(t-\tau) d\tau \\
&= \frac{I_{n,m}}{\gamma_{n,m}\omega_m I_m} R_{n,m}^{(t)} - \frac{\Phi_{n,m}^{(l_n)}}{\gamma_{n,m}\omega_m I_m} \int_0^t m_z^{(\tau)} exp(-\mu_{n,m}\omega_m(t-\tau)) sin\gamma_{n,m}\omega_m(t-\tau) d\tau
\end{aligned} \quad (42)$$

$$\left.\begin{aligned}
&R_{c,m}^{(t)} = \int_0^t \frac{d^2 q_i}{d\tau^2} exp(-\mu_{c,m}\omega_m(t-\tau)) sin\gamma_{c,m}\omega_m(t-\tau) d\tau \quad (c = i, n) \\
&a_{i,j,m} = \frac{1}{2} \sum_{j=1}^{n-1} U_{i,j,m}^{(l_j)} [\frac{A_{t,j,m} f_{i,j,m} - B_{t,j,m} e_{i,j,m}}{e_{i,j,m}^2 + f_{i,j,m}^2} - \frac{A_{t,j,m} g_{i,j,m} + B_{t,j,m} e_{i,j,m}}{e_{i,j,m}^2 + g_{i,j,m}^2}] \\
&b_{i,j,m} = \frac{1}{2} \sum_{j=1}^{n-1} U_{i,j,m}^{(l_j)} [\frac{A_{t,j,m} e_{i,j,m} - B_{t,j,m} g_{i,j,m}}{e_{i,j,m}^2 + g_{i,j,m}^2} - \frac{A_{t,j,m} e_{i,j,m} + B_{t,j,m} f_{i,j,m}}{e_{i,j,m}^2 + f_{i,j,m}^2}] \\
&c_{i,j,m} = \frac{1}{2} [\frac{A_{t,j,m} f_{i,j,m} - B_{t,j,m} e_{i,j,m}}{e_{i,j,m}^2 + f_{i,j,m}^2} + \frac{A_{t,j,m} g_{i,j,m} + B_{t,j,m} e_{i,j,m}}{e_{i,j,m}^2 + g_{i,j,m}^2}] \\
&d_{i,j,m} = \frac{1}{2} [\frac{A_{t,j,m} e_{i,j,m} + B_{t,j,m} f_{i,j,m}}{e_{i,j,m}^2 + f_{i,j,m}^2} - \frac{A_{t,j,m} e_{i,j,m} - B_{t,j,m} g_{i,j,m}}{e_{i,j,m}^2 + g_{i,j,m}^2}] \\
&e_{i,j,m} = \mu_{i,m}\omega_m - \mu_{t,j,m}\omega_{t,j,m}, \quad f_{i,j,m} = \gamma_{i,m}\omega_m + \gamma_{t,j,m}\omega_{t,j,m}, \quad g_{i,j,m} = \gamma_{i,m}\omega_m - \gamma_{t,j,m}\omega_{t,j,m}
\end{aligned}\right\} \quad (43)$$

在上述公式中为了便于区别 i 层地基与 i 段桩的有关量及关系式，特将 i 层地基之 i 改写为 j。函数 $R_{i,m}^{(t)}$ 与 $R_{n,m}^{(t)}$ 通常由数值积分进行计算，它代表地震反应时间函数。由(26)－(31)式和(41)－(43)式有 i 与 n 段桩自由振动与强迫反应时间函数通解分别为：

$$T_{1,m}^{(t)}=T_{x,1,m}^{(t)}+T_{\varphi,1,m}^{(t)}, \quad T_{n,m}^{(t)}=T_{x,n,m}^{(t)}+T_{\varphi,n,m}^{(t)} \tag{44}$$

而地震反应的扭转角位移函数通解为：

$$\varphi_1^{(x,t)}=\sum_{m=1}^{\infty}\varphi_{1,m}^{(x_1,t)}=\sum_{m=1}^{\infty}\Phi_{1,m}^{(x_1)}T_{1,m}^{(t)}, \quad \varphi_n^{(x_n,t)}=\sum_{m=1}^{\infty}\varphi_{n,m}^{(x_n,t)}=\sum_{m=1}^{\infty}\Phi_{n,m}^{(x_n)}T_{n,m}^{(t)} \tag{45}$$

8 解析例题

由(21)—(24)式可解出任意多层地基中桩和地基的频率方程,其它略,仅以1—4层为例求出如下：

8.1 桩基础的频率方程、振型函数及振型系数

8.1.1 贯入一层弹性地基中桩的情况(i=1,n=2)

$$\xi_{1,m}^{(-1)}\alpha_{2,m}tg\,\xi_{1,m}l_1 tg\,\alpha_{2,m}l_2-1=0 \tag{46}$$

$$\Phi_{1,m}^{(x_1)}=\delta_{1,1,m}sin\xi_{1,m}x_1+\delta_{2,1,m}cos\xi_{1,m}x_1 \tag{47}$$

$$\Phi_{2,m}^{(x_2)}=\delta_{1,2,m}sin\alpha_{2,m}x_2+\delta_{2,2,m}cos\alpha_{2,m}x_2 \tag{48}$$

首先取系数$\delta_{1,1,m}=1$对于振动问题并不影响振动形态,因而由(21)、(22)两式可解得各振型系数为：

$$\delta_{1,1,m}=1, \quad \delta_{2,1,m}=0, \quad \delta_{1,2,m}=\xi_{1,m}\alpha_{2,m}^{-1}cos\xi_{1,m}l_1, \quad \delta_{2,2,m}=\delta_{1,2,m}ctg\,\alpha_{2,m}l_2 \tag{49}$$

8.1.2 贯入二层弹性地基中桩的情况(i=1,2;n=3)

$$\xi_{2,m}^{-1}\alpha_{3,m}E_{1,1,m}^{-1}E_{1,2,m}tg\,\alpha_{3,m}l_3+1=0 \tag{50}$$

$$E_{1,1,m}=\xi_{1,m}^{-1}\xi_{2,m}tg\,\xi_{1,m}l_1 tg\,\xi_{2,m}l_2-1$$

$$E_{1,2,m}=\xi_{1,m}^{-1}\xi_{2,m}tg\,\xi_{1,m}l_1+tg\,\xi_{2,m}l_2$$

$$\Phi_{i,m}^{(x_i)}=\delta_{1,i,m}sin\xi_{i,m}x_i+\delta_{2,i,m}cos\xi_{i,m}x_i \quad (i=1,2) \tag{51}$$

$$\Phi_{3,m}^{(x_3)}=\delta_{1,3,m}sin\alpha_{3,m}x_3+\delta_{2,3,m}cos\alpha_{3,m}x_3 \tag{52}$$

$$\delta_{1,1,m}=1, \quad \delta_{2,1,m}=0, \quad \delta_{1,2,m}=\xi_{1,m}\xi_{2,m}^{-1}cos\xi_{1,m}l_1, \quad \delta_{2,2,m}=sin\xi_{1,m}l_1 \tag{53}$$

$$\delta_{1,3,m}=\xi_{2,m}\alpha_{3,m}^{-1}(\delta_{1,2,m}cos\xi_{2,m}l_2-\delta_{2,2,m}sin\xi_{2,m}l_2), \quad \delta_{2,3,m}=\delta_{1,3,m}ctg\,\alpha_{3,m}l_3 \tag{54}$$

8.1.3 贯入三层弹性地基中桩的情况(i=1—3,n=4)

$$\xi_{3,m}^{-1}\alpha_{4,m}E_{1,3,m}^{-1}E_{1,4,m}tg\,\alpha_{4,m}l_4-1=0 \tag{55}$$

$$E_{1,3,m}=E_{1,1,m}+\xi_{2,m}^{-1}\xi_{3,m}E_{1,2,m}tg\,\xi_{3,m}l_3$$

$$E_{1,4,m}=E_{1,1,m}tg\,\xi_{3,m}l_3-\xi_{2,m}^{-1}\xi_{3,m}E_{1,2,m}$$

$$\Phi_{i,m}^{(x_i)}=\delta_{1,i,m}sin\xi_{i,m}x_i+\delta_{2,i,m}cos\xi_{i,m}x_i \quad (i=1,2,3) \tag{56}$$

$$\Phi_{4,m}^{(x_4)}=\delta_{1,4,m}sin\alpha_{4,m}x_4+\delta_{2,4,m}cos\alpha_{4,m}x_4 \tag{57}$$

$$\delta_{1,1,m}=1, \quad \delta_{2,1,m}=0, \quad \delta_{1,2,m}=\xi_{1,m}\xi_{2,m}^{-1}cos\xi_{1,m}l_1, \quad \delta_{2,2,m}=sin\xi_{1,m}l_1 \tag{58}$$

$$\delta_{1,3,m}=\xi_{2,m}\xi_{3,m}^{-1}(\delta_{1,2,m}cos\xi_{2,m}l_2-\delta_{2,2,m}sin\xi_{2,m}l_2), \quad \delta_{2,3,m}=\xi_{1,2,m}sin\xi_{2,m}l_2+\xi_{2,2,m}cos\xi_{2,m}l_2 \tag{59}$$

$$\delta_{1,4,m}=\xi_{3,m}\alpha_{4,m}^{-1}(\delta_{1,3,m}cos\xi_{3,m}l_3-\delta_{2,3,m}sin\xi_{3,m}l_3), \quad \delta_{2,4,m}=\delta_{1,4,m}ctg\,\alpha_{4,m}l_4 \tag{60}$$

8.1.4 贯入四层弹性地基中桩的情况(i=1—4,n=5)

$$\xi_{4,m}^{-1}\alpha_{5,m}E_{1,5,m}^{-1}E_{1,6,m}tg\,\alpha_{5,m}l_5-1=0 \tag{61}$$

$$E_{1,5,m}=E_{2,1,m}-E_{2,2,m}tg\,\xi_{4,m}l_4$$

$$E_{1,6,m}=E_{2,1,m}-tg\,\xi_{4,m}l_4+E_{2,2,m}$$

$$E_{2,1,m}=\xi_{3,m}\xi_{4,m}^{-1}(E_{1,1,m}+\xi_{2,m}^{-1}\xi_{3,m}E_{1,2,m}tg\,\xi_{3,m}l_3)$$

$$E_{2,2,m}=E_{1,1,m}tg\,\xi_{3,m}l_3-\xi_{2,m}^{-1}\xi_{3,m}E_{1,2,m}$$

$$\Phi_{i,m}^{(x_i)}=\delta_{1,i,m}sin\xi_{i,m}x_i+\delta_{2,i,m}cos\xi_{i,m}x_i\quad (i=1,2,3,4) \tag{62}$$

$$\Phi_{5,m}^{(x_5)}=\delta_{1,5,m}sin\alpha_{5,m}x_5+\delta_{2,5,m}cos\alpha_{5,m}x_5 \tag{63}$$

$$\delta_{1,1,m}=1,\quad \delta_{2,1,m}=0,\quad \delta_{1,2,m}=\xi_{1,m}\xi_{2,m}cos\xi_{1,m}l_1,\quad \delta_{2,2,m}=sin\xi_{1,m}l_1 \tag{64}$$

$$\delta_{1,3,m}=\xi_{2,m}\xi_{3,m}^{-1}(\delta_{1,2,m}cos\xi_{2,m}l_2-\delta_{2,2,m}sin\xi_{2,m}l_2),\quad \delta_{2,3,m}=\delta_{1,2,m}sin\xi_{2,m}l_2+\delta_{2,2,m}cos\xi_{2,m}l_2 \tag{65}$$

$$\delta_{1,4,m}=\xi_{3,m}\xi_{4,m}^{-1}(\delta_{1,3,m}cos\xi_{3,m}l_3-\delta_{2,3,m}sin\xi_{3,m}l_3),\quad \delta_{2,4,m}=\delta_{1,3,m}sin\xi_{3,m}l_3+\delta_{2,3,m}cos\xi_{3,m}l_3 \tag{66}$$

$$\delta_{1,5,m}=\xi_{4,m}\alpha_{5,m}(\delta_{1,4,m}cos\xi_{4,m}l_4-\delta_{2,4,m}sin\xi_{4,m}l_4),\quad \delta_{2,5,m}=\delta_{1,5,m}ctg\,\alpha_{5,m}l_5 \tag{67}$$

8.2 地基的频率方程、振型函数及振型系数

8.2.1 一层弹性地基的情况(i=n−1=1)

$$cos\theta_{1,m}l_1=0,\quad \theta_{1,m}=\frac{m\pi}{2l_1}\quad (m=1,3,5,\cdots\cdots) \tag{68}$$

$$\Psi_{1,m}^{(x_1)}=d_{1,1,m}sin\theta_{1,m}x_1+d_{2,1,m}cos\theta_{1,m}x_1 \tag{69}$$

对于动力问题，取$d_{1,1,m}=1$并不影响振动形态，因而由(23)和(24)两式可解得各系数为：

$$d_{1,1,m}=1,\quad d_{2,1,m}=0 \tag{70}$$

8.2.2 二层弹性地基的情况(i=1,n−1=2)

$$\theta_{1,m}^{-1}\theta_{2,m}tg\,\theta_{1,m}l_1tg\,\theta_{2,m}l_2-1=0 \tag{71}$$

$$\Psi_{i,m}^{(x_i)}=d_{1,i,m}sin\theta_{i,m}x_i+d_{2,i,m}cos\theta_{i,m}x_i\quad (i=1,2) \tag{72}$$

$$d_{1,1,m}=1,\quad d_{2,1,m}=0,\quad d_{1,2,m}=\theta_{1,m}\theta_{2,m}^{-1}cos\theta_{1,m}l_1,\quad d_{2,2,m}=d_{1,2,m}ctg\,\theta_{2,m}l_2 \tag{73}$$

8.2.3 三层弹性地基的情况(i=1,2;n−1=3)

$$F_{1,1,m}^{-1}F_{1,2,m}tg\,\theta_{3,m}l_3+1=0 \tag{74}$$

$$F_{1,1,m}=\theta_{1,m}^{-1}\theta_{2,m}tg\,\theta_{1,m}l_1tg\,\theta_{2,m}l_2-1$$

$$F_{1,2,m}=\theta_{2,m}^{-1}\theta_{3,m}(\theta_{1,m}^{-1}\theta_{2,m}tg\,\theta_{1,m}l_1+tg\,\theta_{2,m}l_2)$$

$$\Psi_{i,m}^{(x_i)}=d_{1,i,m}sin\theta_{i,m}x_i+d_{2,i,m}cos\theta_{i,m}x_i\quad (i=1,2,3) \tag{75}$$

$$d_{1,1,m}=1,\quad d_{2,1,m}=0,\quad d_{1,2,m}=\theta_{1,m}\theta_{2,m}^{-1}cos\theta_{1,m}l_1,\quad d_{2,2,m}=sin\theta_{1,m}l_1 \tag{76}$$

$$d_{1,3,m}=\theta_{2,m}\theta_{3,m}^{-1}(d_{1,2,m}cos\theta_{2,m}l_2-d_{2,2,m}sin\theta_{2,m}l_2),\quad d_{2,3,m}=d_{1,3,m}ctg\,\theta_{3,m}l_3 \tag{77}$$

8.2.4 四层弹性地基的情况(i=1−3;n−1=4)

$$\theta_{3,m}^{-1}\theta_{4,m}F_{1,3,m}^{-1}F_{1,4,m}tg\,\theta_{4,m}l_4-1=0 \tag{78}$$

$$F_{1,3,m}=F_{1,1,m}+F_{1,2,m}tg\,\theta_{3,m}l_3,\quad F_{1,4,m}=F_{1,1,m}tg\,\theta_{3,m}l_3-F_{1,2,m}$$

$$\Psi_{i,m}^{(x_i)}=d_{1,i,m}sin\theta_{i,m}x_i+d_{2,i,m}cos\theta_{i,m}x_i\quad (i=1,2,3,4) \tag{79}$$

$$d_{1,1,m}=1,\quad d_{2,1,m}=0,\quad d_{1,2,m}=\theta_{1,m}\theta_{2,m}^{-1}cos\theta_{1,m}l_1,\quad d_{2,2,m}=sin\theta_{1,m}l_1 \tag{80}$$

$$d_{1,3,m}=\theta_{2,m}\theta_{3,m}^{-1}(d_{1,2,m}cos\theta_{2,m}l_2-d_{2,2,m}sin\theta_{2,m}l_2),\ d_{2,3,m}=d_{1,2,m}sin\theta_{2,m}l_2+d_{2,2,m}cos\theta_{2,m}l_2 \tag{81}$$

$$d_{1,4,m}=\theta_{3,m}\theta_{4,m}^{-1}(d_{1,3,m}cos\theta_{3,m}l_3-d_{2,3,m}sin\theta_{3,m}l_3),\quad d_{2,4,m}=d_{1,4,m}ctg\,\theta_{4,m}l_4 \tag{82}$$

9 讨论与分析

(1)若桩整体沿长度方向为等截面、等剪切模量、等抗扭刚度与等质量密度和等惯性矩，则有：

$$G_iJ_i=G_nJ_n=GJ,\quad \rho_{pi}J_{pi}=\rho_{pn}J_{pn}=\rho_pJ_p \tag{83}$$

$$\zeta_t^2=\zeta_m^2=\zeta^2=\frac{GJ}{\rho_p J_p} \tag{84}$$

有均质桩自振圆频率ω_m、固有频率f_m与周期$T_{G,m}$为：

$$\omega_m=\xi_{i,m}\sqrt{1+\frac{K_\varphi}{\xi_{i,m}^2 GJ}}\sqrt{\frac{GJ}{\rho_p J_p}},\quad f_m=\frac{\omega_m}{2\pi},\quad T_{G,m}=\frac{2\pi}{\omega_m} \tag{85}$$

若桩为圆形截面，则$J=J_p$，(85)式便化简为：

$$\omega_m=\xi_{i,m}\sqrt{1+\frac{K_\varphi}{\xi_{i,m}^2 GJ}}\sqrt{\frac{G}{\rho_p}},\quad f_m=\frac{\omega_m}{2\pi},\quad T_{G,m}=\frac{2\pi}{\omega_m} \tag{86}$$

(2)关于弹性地基，自振圆频率$\omega_{t,i,m}$、固有频率$f_{i,m}$与周期$T_{g,m}$分别为：

$$\omega_{t,i,m}=\theta_{i,m}\eta_i=\theta_{i,m}\sqrt{\frac{G_{T,i}J_{T,i}}{\rho_{t,i}J_{t,i}}},\quad f_{i,m}=\frac{\omega_{t,i,m}}{2\pi},\quad T_{g,m}=\frac{2\pi}{\omega_{t,i,m}} \tag{87}$$

当地基为单位圆形截面的模型时，则有$J_{t,i}=J_{T,i}$，(87)式便简化为：

$$\omega_{t,i,m}=\theta_{i,m}\sqrt{\frac{G_{T,i}}{\rho_{t,i}}},\quad f_{i,m}=\frac{\omega_{t,i,m}}{2\pi},\quad T_{g,m}=\frac{2\pi}{\omega_{t,i,m}} \tag{88}$$

10 结束语

本文分别给出了桩基础与地基的耦合扭转地震反应基本振动方程(7)，桩的扭转地震反应基本振动方程(8)，分层弹性地基场地土的旋转地震反应波动方程(9)，自由扭转振动方程(13)－(15)，条件式(19)、(20)，频率矩阵式(21)－(24)，振型系数与振型函数及时间函数解(25)－(31)式，强迫扭转地震反应解(32)－(43)式，自由扭转振动与强迫扭转反应时间函数通解(44)式及角位移函数通解(45)式，解析例题，等截面匀质桩体自由扭转振动圆频率与固有频率及周期(85)，分层弹性地基的圆频率与固有频率及周期(87)、(88)式。上述解析公式，对分层弹性地基中单桩基础扭转地震反应问题进行了全面细致的分析。文中的土力学参数要根据土力学或土动力学的有关书籍查取，当然现场实测更好。地震扭转参数要根据地震观察或实测结果取值。本文充分体现了桩基础扭转地震反应的力学特性及受力特点，便于分析与理解，为单桩基础的扭转地震反应分析提供了一种新的解析方法，并为多层与高层建筑结构以及海洋与海岸工程的群桩基础扭转地震反应分析提供了有利条件。

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THE TORSIONAL SEISMIC RESPONSE ANALYSIS FOR THE PILE FOUNDATION UNDER THE ACTION OF TORSIONAL WAVE OF THE LAYERED ELASTO-FOUNDATION SOIL

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Abstract

In this paper, the reasonable mechanics model are established for single pile foundation in layered elastic ground by means of characteristic analysis. Then the torsional vibration analysis of the pile is carried out according to layered elasto-foundation soil model, thus the analytic solutions for the torsional free vibration characteristics and the coercion responses under the action of the torsional seismic loading and torsional vibration loading of the pile are given. The analytic formulas in the paper provide a new analytic method for the torsional seismic response analysis of the pile foundation in layered elastic ground.

Subject words: Torsional wave, Seismic response analysis, Foundation, Pile

会讯

1995年甘肃省科技期刊主编新春座谈会在兰召开

1995年元月6日，甘肃省科委和省新闻出版局在兰州联合召开了“1995年甘肃省科技期刊主编新春座谈会”。出席会议的有中央在甘和本省80多家科技期刊的96名代表，省内外一些关心支持科技期刊事业的企事业单位也派人参加了会议。这次会议得到了天津市天磁公司和兰州八一兴隆矿泉饮料总厂等单位的大力协助。

会上围绕“加强科技期刊管理工作，深化科技期刊改革和繁荣科技期刊事业”的主题，省科委副主任吴新科和省新闻出版局副局长杨效知同志作了重要讲话。科技期刊主编和特邀企业的代表也进行了大会发言。

与会代表们认为：科技期刊是科技工作的重要组成部分，她孕育着丰富的信息资源。这次会议的召开，对沟通期刊与企业的联系，增加期刊与企业的合作机会，促进各刊编辑部以期刊为阵地，以科技为依托，以信息为媒介，充分利用期刊联系广泛的优势，大力开展科技咨询、成果推广和技术培训等工作将起到积极的推动作用。

（期宣）