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INERTIAL MODE OF MHD WAVE IN THE EARTH'S CORE

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Abstract

The outer core of the earth, being in the plasma state with high temperature, high pressure, high density, and immersed in a strong toroidal magnetic field, can support varieties of plasma waves. Under influene of earth rotation, MHD wave propagates alone field lines in two modes; eastward inertal mode and westward magnetic mode, both are characterized dy high dispersion. The magnetic mode is related with the secularvariation and westward shift of the main magnetic field. The inertial mode has periods of the order of days, its magnetic effects would be shielded from the earth's surface dy conductive mantle. However, the mechanical couplins between the core and the lower mantle would be possidle, hence a periodic mechanic process would de produced by MHD wave in the earth's core。

Introduction

On the basis of geophysical and geochemical data, the outer core of the earth is in the liquid state with high temprature, high pressure, highdensity, Studies on the earth magnetism imply that the main geomagnetic field arises in such a liquid, metallic core, where both dynamic and electromagnetic processes are coupled each other. Therefore, hydromagnetic dynamics equations should be used to describe processes in the earth's core。

As early as 1940's and 1950's Bullard (1949,1955), Bullard and Gellman (1954), Elsasser(1946,1956) Backus (1958) and Herzenberg (1958) extensively stndied the MHD origin of the main magnetic field and its secular variation (S.V.). After 1960's, Hide (1966 a, b), Crossley and Smylie (1975), and others attempted to explain the generation of the earth's magnetic field and its secular variation by the use of MHD wave propagating in the earth's core. They focused to be very long period waves (years or centuries) since short period variations wonld be suppressed from the magnetic record at the earth's surface by the high conductive mantle.

However, the mechanic coupling on or/and near the core mantle interface may propagate up to the earth's surface and exhibit itself in varieties of geophysical fields. This perhaps gives us a plausible clue to understanding some global periodic and rhythmical phenomena.

In this paper, we discuss a special form of MHD equation suitable for the outer core, deducing dispersion equation of the MHD wave obtaining fomulas of the phase and group velocities for spatial wave length of $m=1\sim5$ and $n=1\sim15$. we are specially interested in those waves with periods of a few days to tens days, which might be connected with dynamic processes in the earth.

Model and basic equations

The model used in this paper is shown in Fig. 1. Between the solid mantle and the inner core there is a liquid outer core with the outer radius R_a and the inner radius R_a Densities (ρ) and conductivities (σ) in these three regions are indicated dy subscripts a, b, and c, respectively as depicted in Fig. 1. In the figure is also shown a local cartesian frame (x, y, z), whose origin is located at the mean colatitude θ of the region under consideration, and the x, y and z axes are directed eatward, northward and upward, respectively.

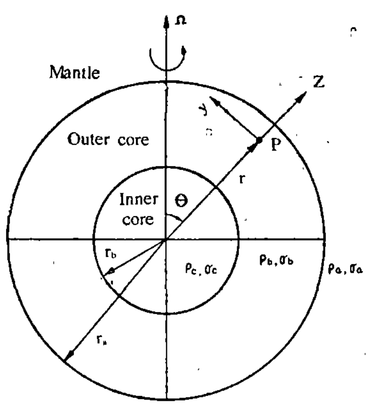

Fig. 1

Following assumptions are addopted:

1. The outer core is an invicid, incompressible, homogeneous liquid
2. The outer core is perfectly conductive.

These assumptions are good approximations in present work, since the time scale we are interested in is mnch less than the diffusion time of the core's magnetic field (~30000years).

The equations governing the processes of the core in the corotational frame are as follows:

$$\frac{D\vec{V}}{Dt}+2\vec{\Omega}\times\vec{V}=-\frac{1}{\rho}\nabla p+\frac{1}{\rho}\vec{j}\times\vec{B} \qquad (1)$$

$$\nabla\times\vec{B}=\mu\vec{j} \qquad (2)$$

$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \qquad (3)$$

$$\nabla\cdot\vec{B}=0 \qquad (4)$$

$$\vec{E}=-\vec{V}\times\vec{B} \qquad (5)$$

$$\nabla\cdot\vec{V}=0 \qquad (6)$$

where
$$\frac{D}{Dt}\equiv\frac{\partial}{\partial t}+(\vec{V}\cdot\nabla) \qquad (7)$$

It is difficult to solve these equations in general form, and simplifications are necessary. In this paper a toroidal motion is assumed near the surface of the outer core, e.g. radial component of the velocity is ignored.

In the local Cartesian frame

$$\begin{aligned}\vec{j}\times\vec{B}&=\frac{1}{\mu}(\nabla\times\vec{B})\times\vec{B}\\
&=\hat{X}\left[-\frac{1}{2\mu}\frac{\partial B_Z^2}{\partial X}-\frac{B_Z}{\mu}\left(\frac{\partial B_Y}{\partial X}-\frac{\partial B_X}{\partial Y}\right)\right]\\
&+\hat{Y}\left[-\frac{1}{2\mu}\frac{\partial B_Z^2}{\partial Y}+\frac{B_X}{\mu}\left(\frac{\partial B_Y}{\partial X}-\frac{\partial B_X}{\partial Y}\right)\right]\\
&+\hat{Z}\left[\frac{B_Y}{\mu}\frac{\partial B_Z}{\partial Y}+\frac{B_X}{\mu}\frac{\partial B_Z}{\partial X}\right]\end{aligned} \qquad (8)$$

$$\vec{\Omega}\times\vec{V}=\Omega(-\hat{X}V\cos\theta+\hat{Y}u\sin\theta-\hat{Z}\sin\theta) \qquad (9)$$

Substituting equations (8) and (9) into (1) and eliminating $(p+B_Z^2/2\mu)$, we find that

$$\frac{D\zeta}{Dt}+2\Omega V\frac{\sin\theta}{r}=\frac{1}{\mu\rho}\left(B_x\frac{\partial}{\partial X}+B_y\frac{\partial}{\partial Y}\right)\left(\frac{\partial B_y}{\partial X}-\frac{\partial B_x}{\partial Y}\right) \qquad (10)$$

where
$$\zeta\equiv\frac{\partial V}{\partial X}-\frac{\partial u}{\partial Y} \qquad (11)$$

Combination of equations (3) and (5) leads to

$$\frac{\partial\vec{B}}{\partial t}=\nabla\times(\vec{V}\times\vec{B}) \qquad (12)$$

or
$$\frac{D\vec{B}}{Dt}=(\vec{B}\cdot\nabla)\vec{V} \qquad (13)$$

The components of equation (13) can be written as follows

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial X}+V\frac{\partial}{\partial Y}\right)B_x=\left(B_x\frac{\partial}{\partial X}+B_y\frac{\partial}{\partial Y}\right)u$$

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial X}+V\frac{\partial}{\partial Y}\right)B_Y=\left(B_x\frac{\partial}{\partial X}+B_Y\frac{\partial}{\partial Y}\right)V$$

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial X}+V\frac{\partial}{\partial Y}\right)B_z=0 \tag{14}$$

On introducing Coriolis parameter, $f=2\Omega\cos\theta$, in the form

$$f=f_0+\beta y \tag{15}$$

where

$$f_0=2\Omega\cos\theta_0,\quad \beta=2\Omega\sin\theta_0/Ra \tag{16}$$

equation (10) can be further simplified as follows

$$\left[\frac{D^2}{Dt^2}-\frac{1}{\mu\rho}\left(B_X\frac{\partial}{\partial X}+B_Y\frac{\partial}{\partial Y}\right)^2\right]\nabla_H^2\zeta+\beta\frac{D}{Dt}\frac{\partial\zeta}{\partial X}=0 \tag{17}$$

Now, let's consider the behaviour of a small-amplitude disturbance, superimposed on a uniform basic flow U_0 in the x direction and a background magnetic field of uniform strength B_0 at an angle α with the x direction, thus

$$\begin{aligned} u&=U_0+u_1\\ v&=v_1\\ B_X&=B_0\cos\alpha+b_X\\ B_Y&=B_0\sin\alpha+b_Y\\ B_Z&=b_Z \end{aligned} \tag{18}$$

The disturbance is assumed to vary harmonically in space and time, e.g.

$$\{u_1,v_1,\zeta_1,b_X,b_Y\}=\{\bar{u},\ \bar{v},\ \bar{\zeta},\ \bar{b}_X,\ \bar{b}_Y\}\exp i(kx+ly-\hat{\omega}t) \tag{19}$$

The required dispersion relationship can de obtained

$$\omega^2+\frac{\beta k}{k^2+l^2}\omega-v_A^2(R\cos\alpha+l\sin\alpha)^2=0 \tag{20}$$

where V_A is Alfven velocity

$$V_A=\frac{B_0}{\sqrt{\mu\rho}} \tag{21}$$

and

$$\omega=\hat{\omega}-U_0k \tag{22}$$

The equation (20) has two roots

$$\omega_{i;m}=-\frac{\beta k}{2(k^2+l^2)}\left\{1\pm\sqrt{1+\left[\frac{2v_A(k^2+l^2)(k\cos\alpha+l\sin\alpha)}{\beta k}\right]^2}\right\} \tag{23}$$

where subscripts i and m stand for inertial and magnetio modes, respecitively.

From the dispersion relationship (23), the phase and group velocities can be obtained (relative to the basic flow)

$$(V_{i;m})_x = \frac{\omega_{i,m}}{k} = -\frac{\beta}{2(k^2+l^2)}\left\{1 \pm \sqrt{1+\left[\frac{2v_A(k^2+l^2)(k\cos\alpha+l\sin\alpha)}{\beta k}\right]^2}\right\} \tag{24}$$

$$(V_{i;m})_y = \frac{\omega_{i;m}}{l} = -\frac{\beta k}{2l(k^2+l^2)}\left\{1 \pm \sqrt{1+\left[\frac{2v_A(k^2+l^2)(k\cos\alpha+l\sin\alpha)}{\beta k}\right]^2}\right\} \tag{25}$$

$$(U_{i;m})_x = \frac{1}{2\omega_{i;m}+\frac{\beta k}{k^2+l^2}}\left[2v_A^2(k\cos\alpha+l\sin\alpha)\cos\alpha+\frac{\beta\omega_{i;m}(k^2-l^2)}{(k^2+l^2)^2}\right] \tag{26}$$

$$(U_{i;m})_y = \frac{1}{2\omega_{i;m}+\frac{\beta k}{k^2+l^2}}\left[2v_A^2(k\cos\alpha+l\sin\alpha)\sin\alpha+\frac{2\omega_{i;m}\beta kl}{(k^2+l^2)^2}\right] \tag{27}$$

If the density ρ, the background magnetic field B_0 and the angle α are specified, the frequencies, phase and group velocities of the disturbance can be calculated for spatial wave numbers k and L.

Magnetic field in the earth's core

The magnetic field in the earth's core can be divided its poloidal and toroidal darts, B_P Only B_T and B_P has a radial component, with lines of force passing out of the core, through the mantle, and on into space. The toroidal magnetic field B_T, if it exists, has no radial component and thus confined to its region of origin, the core. Extrapolation of the field at the earth's surface leads to information about B_P at the core mantle interface B_T, on the other hand, cannot be detected directly.

According to the concept of frozen-in field in plasma and the fact of shearing rotation of the earth, B_T in the core is genrally regarded as the result of the interaction of B_P with zonal shearing motion. The ratio B_T/B_P is rouphly of the order of a magnetic Reynolds number R_m

$$R_m = \mu\sigma LV \tag{28}$$

In the earth' s core R_m is about 6000 (Hide, 1966). In spite of uncertainties in estimation of R_m, it is very likely that $B_T \gg B_P$. In this paper a simple zonal magnetic field model is used, that is

$$B_0 = 0.01\,\mathrm{Wb/m^2} \tag{29}$$

$$\alpha = 0$$

in this special case, equations (23) to (27) are further simplified further as follownig.

$$\omega_{i;m} = -\frac{\beta k}{2(k^2+l^2)}\left\{1 \pm \sqrt{1+\left[\frac{2v_A(k^2+l^2)}{\beta}\right]^2}\right\} \tag{30}$$

$$(V_{i;m})_X=\frac{\beta}{2(k^2+l^2)}\left\{1\pm\sqrt{1+\left[\frac{2v_A(k^2+l^2)}{\beta}\right]^2}\right\} \quad (31)$$

$$(V_{i,m})_Y=-\frac{\beta k}{2l(k^2+l^2)}\left\{1\pm\sqrt{1+\left[\frac{2v_A(k^2+l^2)}{\beta}\right]^2}\right\} \quad (32)$$

$$(U_{i;m})_X=\frac{1}{2\omega_{i;m}+\frac{\beta k}{k^2+l^2}}\left[2v_A^2k+\frac{\beta\omega_{i,m}(k^2-l^2)}{(k^2+l^2)^2}\right] \quad (33)$$

$$(U_{i;m})_Y=\frac{2\omega_{i;m}\beta kl}{2\omega_{i,m}(k^2+l^2)^2+\beta k(k^2+l^2)} \quad (34)$$

Characteristios of inertial mode

It is seen from equations (30) to (34) that both magngtic and inertial modes are highly dispersive. Their angular frequencies, phase and group velocities depend on the spatial structure of the disturbance.

In analysing geomagnetic field and other geophysical fields, it is conventional to use spherical harmonic series

$$\phi(r,\theta,\lambda)=R_0\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\left(\frac{R_0}{r}\right)^{n+1}(g_n^m\cos m\lambda+h_n^m\sin m\lambda)p_n^m(\cos\theta) \quad (35)$$

where R_0 is the radior of the earth, p_n^m ‘$\cos\theta$) are the Schmidt quasinormalized dolynomials of argument $\cos\theta$, andy$_n^m$ and h_n^m are Gauss coefficients. The wave numbers k and L introduced above are related to m and n as follows,

$$m=kR_a\sin\theta \qquad (36)$$
$$n-m=lR_a$$

The magnetic mode has oscillation periods comparable with the time scale of the secular variation of the geomagnetic field (Hide, 1966). The inertial mode, on the other hand, has periods of the order of days. It is this mode that might contribute to some global periodic or rhythmical phenomena.

In Table 1 to 4, the frequencies, periods, group velocities and propagation times around the earth are listed for inertial modes of MHD waves with various spatial structures indicated by m and n.

Excitation of oscillations in the earth's core and their manifestations on the earth's surface

In spite of lack of knowledge on the earth's core,it is very likely that the rotation is one of factors governing the core movement, which makes the large—scale convection in the core to exhibit highly anisotropic ly.

Another plausible speculation is that a toroidal magnetic field is much stronger than poloidal one in the earth's core. In such a hydromagnetic medium, there are varieties of eddies with highly different spatial sizes. The energy can be transferred from small-scale eddies to large-scale ones or vice versa.

The disturbances in the earth's core may be excicted by earthqakes, sudden changes of the earth rotation and other events. As long as a disturbance is excited near the core-mantle interface, each of the spatial components of the disturbance will propagate with corresponding period around the earth's core. Since tie MHD waves carry their energy in two forms: mechanical and electromagnetic, the interaction (or coupling) between the core and the mantle is associated with both mechanical and electomagnetic energy transfer from the core to the mantle or/and vice versa.

As for inertial mode of MHD waves mentioed above, electromagnetic energy cannot be transferred upward too far because of the electrically conductive mantle and relatively short periods. Mechanical energy, might be transferred upto the upper mantle and even the ground surface, and exhibit itself in some geophysical fields and phenomena.

It shoud be pointed out that model used in this paper is simpified one, therefore the results are only qualitatve.

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Table 1. Frequencies of inertial mode of MHD waves
for different spatial structure (r and/s)

n \ m	1	2	3	4	5
1	9.622				
2	5.703	4.811			
3	2.566	4.106	3.207		
4	1.339	2.851	2.980	2.405	
5	0.802	1.889	2.457	2.206	1.924
6	0.529	1.283	1.901	2.035	1.873
7	0.374	0.908	1.444	1.735	1.734
8	0.277	0.670	1.102	1.426	1.543
9	0.214	0.511	0.855	1.160	1.337
10	0.170	0.401	0.676	0.945	1.141
11	0.138	0.323	0.545	0.775	0.967
12	0.114	0.265	0.446	0.642	0.820
13	0.096	0.221	0.373	0.537	0.697
14	0.082	0.187	0.313	0.454	0.596
15	0.071	0.160	0.267	0.388	0.513

Table 2. the same as Table 1, but for periods (days)

n \ m	1	2	3	4	5
1	0.8				
2	1.3	1.5			
3	2.8	1.8	2.3		
4	5.4	2.6	2.4	3.0	
5	9.1	3.8	3.0	3.2	3.8
6	13.7	5.7	3.8	3.5	3.9
7	19.5	8.0	5.0	4.2	4.2
8	26.2	10.9	6.6	5.1	4.7
9	34.0	14.2	8.5	6.3	5.4
10	42.8	18.1	10.8	7.7	6.4
11	52.7	22.5	13.3	9.4	7.5
12	63.6	27.5	16.3	11.3	8.9
13	75.6	32.9	19.6	13.5	10.4
14	88.5	38.9	23.2	16.0	12.2
15	102.6	45.5	27.2	18.7	14.2

Table 3. the same as Table 1, but for group velocities (m/s)
(+ and − for westward and eastward
propagations, respectivey)

n m	1	2	3	4	5
1	279.2				
2	30.7	69.8			
3	-34.7	42.1	31.0		
4	-28.0	7.7	24.7	17.5	
5	-19.4	-5.9	12.6	15.4	11.2
6	-13.7	-8.7	3.4	10.5	10.3
7	-10.0	-8.2	-1.4	5.6	8.1
8	-7.6	-7.0	-3.3	1.9	5.4
9	-5.9	-5.8	-3.9	-0.3	3.0
10	-4.8	-4.8	-3.8	-1.5	1.2
11	-3.7	-4.1	-3.5	-2.0	0.0
12	-3.2	-3.4	-3.1	-2.2	-0.7
13	-2.7	-2.9	-2.8	-2.2	-1.1
14	-2.3	-2.5	-2.4	-2.1	-1.3
15	-2.0	-2.2	-2.2	-9.1	-1.4

Table 4. the same as Table 1 but for propagation times around the earth's core (days)
(+ and −for westward and eastward)
propagations, respectively)

n m	1	2	3	4	5
1	0.9				
2	8.3	3.6			
3	-7.3	6.0	8.2		
4	-9.1	33.2	10.3	14.9	
5	-13.1	-43.3	20.1	16.6	22.8
6	-18.6	-29.3	-74.7	24.2	24.7
7	-25.4	-31.0	-082.6	45.7	31.5
8	-33.5	-36.3	-76.4	132.8	47.1
9	-42.9	-43.6	-65.9	-848.9	84.4
01	-53.6	-52.5	-67.3	-173.1	207.5
11	-65.4	-62.8	-73.2	-127.3	8770.2
12	-78.6	-74.5	-81.7	-117.2	-361.7
13	-93.0	-87.5	-92.2	-118.0	-228.5
14	-108.7	-101.8	-104.4	-124.1	-193.4
15	-125.6	-117.3	-118.1	-133.5	-183.1

地核中MHD波的惯性模

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摘　　要

地球外核处于高温、高压、高密度的等离子体状态，可以支持多种磁流体波的激发和传播。在地核中存在强的环型磁场，沿磁力传播的MHD波受到地球旋转的影响，分成东行的惯性模和西行的磁模，这两种波模具有高度色散的特点。其中西行磁模波与地磁场长期变化和西漂有关。东行惯性模波具有数天到数百天的周期，其磁效应被导电地幔所屏蔽，传不到地表，但它与下地幔物质可能发生力学耦合，在地幔中产生周期变化的力学过程，这种影响可能达及地表并在地表的地球物理现象中表现出来，它可能是某些周期性或韵律性地球物理现象的原因之一。